VOLUME 26 AUGUST, 1938 NUMBER 8

PROCEEDINGS
of
The Institute of Radio Engineers

Application Blank for Associate Membership on Page IX

Institute of Radio Engineers Forthcoming Meetings

PACIFIC COAST CONVENTION
Portland, Oregon
August 10 and 11, 1938

CLEVELAND SECTION
September 22, 1938

DETROIT SECTION
September 16, 1938

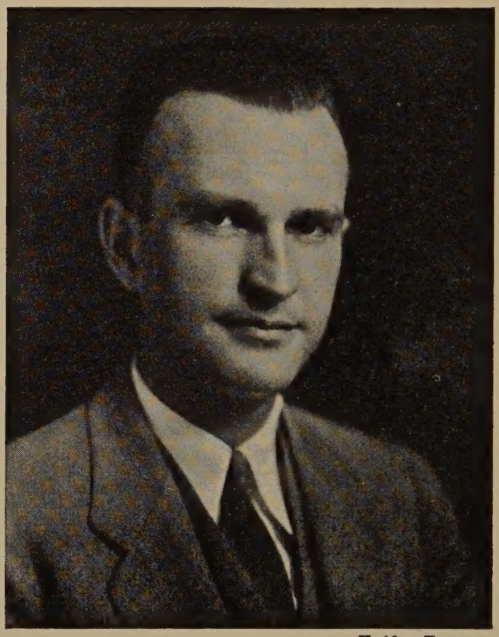
LOS ANGELES SECTION
September 20, 1938

NEW YORK MEETING
October 5, 1938

PHILADELPHIA SECTION
September 1, 1938

PITTSBURGH SECTION
September 20, 1938

WASHINGTON SECTION
September 12, 1938



Kaiden-Keystone

John C. Warner
1896 - 1938

John Chester Warner was born in Freeport, Illinois, on February 14, 1896. He received an A.B. degree from Washburn College in 1917, an A.M. from the University of Kansas in 1921, and an M.S. degree in electrical engineering from Union College in 1925.

He served in the United States Signal Corps from 1917 to 1919 as a radio officer. The next year was spent at the University of Kansas an an instructor, followed by a few months as an assistant physicist with the Bureau of Standards. From 1920 to 1931 he did research work on vacuum tubes at the General Electric Company laboratories in Schenectady, New York, becoming assistant head of the vacuum-tube engineering department.

In 1931 he joined the RCA Radiotron Company as manager of the research and development laboratory, becoming vice president in 1932 and general manager also in 1934.

He joined the Institute as an Associate in 1919, transferred to Member in 1926, and to Fellow in 1933.

Mr. Warner was killed in an automobile accident which occurred while he was driving to work on the morning of July 21, 1938.

INSTITUTE NEWS AND RADIO NOTES

Thirteenth Annual Convention

Our Thirteenth Annual Convention, held in New York City, broke all previous records with an attendance of 1766 men and 100 women. This is about fifty per cent larger than any previous convention attendance. The forty-nine papers listed and summarized in the June PROCEEDINGS were all presented as scheduled. There were twenty-nine exhibitors.

As indicated in the program, the Institute Medal of Honor, Morris Liebmann Memorial Prize, and PROCEEDINGS Paper Prize were presented to their recipients during the technical session on the evening of June 16. Presentations were made by President Pratt.

The boat trip which took the place of the annual banquet was attended by about 275.

Conference on Volume Indicators and Reference Volume

For some months, representatives of the Bell Telephone Laboratories, Columbia Broadcasting System, and the National Broadcasting Company have been co-operating in a project to establish a standard instrument for the indication of the level of program material being transmitted over wire lines and handled by equipment employed in broadcasting. It was felt that developments had proceeded to the point where a larger group of those interested in broadcasting and other branches of radio should be invited to participate in a conference on the subject. This conference was held in the Hotel Pennsylvania on June17.

The three original groups were represented by: Bell Telephone Laboratories—H. A. Affel, S. Brand, S. Duma, D. K. Gannett, and Iden Kerney; Columbia Broadcasting System—R. A. Bradley and H. A. Chinn; and National Broadcasting Company—W. M. Baston, H. C. Luttgens, G. O. Milne, R. M. Morris, and George Nixon. The American Telephone and Telegraph Company was represented by W. E. Bloecker, F. A. Cowan, V. E. Love, A. F. Rose, and N. R. Weible. The following lists the names of others who were present: H. P. Westman, acting chairman; R. W. Armstrong, L. F. Curtis, E. T. Dickey, D. E. Foster, D. W. Gellerup, F. M. Greene, A. Hass, L. C. F. Horle, J. J. Keel, J. J. Long, J. H. Miller, H. L. Olesen, E. G. Pack, C. F. Quentin, C. D. Samuelson, S. H. Simpson, Charles Singer, A. E. Thiessen, L. P. Tuckerman, J. D. Wallace, L. P. Wheeler, J. Wright, and N. J. Zehr.

R. M. Morris of the National Broadcasting Company presented first a brief history of the development of volume indicators and pointed out a number of features which were considered undesirable in the design and construction of various instruments that have been used for this purpose. He then pointed out that the Bell Telephone Laboratories, Columbia Broadcasting System, and National Broadcasting Company had been collaborating with the Weston Electrical Instrument Corporation with the objective of devising an instrument which would be an improvement over the existing equipment. The design of this new instrument and its performance was then outlined and some high-speed motion pictures comparing its performance with that of the present "standard" instrument were shown. This was followed by a demonstration prepared by the Bell Telephone Laboratories showing a number of different instruments which have been and are being used and the proposed new model. All of the comments made on the performance of the new instrument were favorable.

The calibration of the device and its influence on those active in other branches of radio engineering were then discussed. There appeared to be no strong opinions as to whether the instrument be calibrated on a 500- or 600-ohm basis to read zero at either 6 or 10 milliwatts, or perhaps 1 milliwatt, of single-frequency sine-wave input. It was felt that the decisions on these points could well be left to the original committee. In reaching the decisions, the committee will take into account the opinions expressed at the international conference recently held at Oslo, as well as the opinions expressed in this country.

Committee Work

Sections and Membership Committees

A joint meeting of the Sections Committee and the Membership Committee was held during the Institute Convention on June 16 in the Hotel Pennsylvania, New York City. Those present were E. D. Cook, chairman of the Sections Committee, presiding: C. T. Burke, H. L. Byerlay, W. F. Cotter, B. V. K. French, R. T. Gabler, Ben Kievit, Jr., H. S. Knowles, J. H. Miller, D. E. Noble, R. C. Poulter, E. H. Rietzke, G. T. Royden, E. R. Sanders, H. J. Schrader, A. R. Taylor, F. E. Terman, C. F. Wolcott, and L. C. Young of the Sections Committee, and C. E. Scholz, chairman; H. A. Chinn, I. S. Coggshall, F. W. Cunningham, H. C. Gawler, R. M. Heintz, L. G. Pacent, F. X. Rettenmeyer, and Bernard Salzberg of the Membership Committee. In addition, Haraden Pratt, president; H. P. Westman, secretary; and J. D. Crawford, assistant secretary, attended the meeting. The follow-

ing sections were represented: Boston, Chicago, Connecticut Valley, Detroit, Emporium, Indianapolis, Philadelphia, Pittsburgh, Rochester, San Francisco, Seattle, Toronto, and Washington.

Data on section membership, meetings, and finances were first considered. Although discussed in detail, no action was considered necessary in view of the successful manner in which our sections are progressing.

A lengthy discussion was held as to the desirability of improving our relations with the various colleges which offer communication courses. It was felt that many more students could be enrolled as members if the activities of the Institute could be brought more forcibly to their attention. Methods of bringing about improved contact with qualified students were discussed.

The desirability of qualified associates transferring to Member grade was discussed. It was felt that this problem might well be attacked by the officers of the sections who could encourage men qualified for Member grade to apply and thus assume their proper standing in the Institute.

Some of the major proposed changes in the Institute constitution which will be placed before the membership for ballot this year were discussed.

Technical Committee on Electronics

A meeting of the Technical Committee on Electronics of the Institute was held in the Hotel Pennsylvania, New York City, on June 16, 1938. Those present were P. T. Weeks, chairman; R. S. Burnap, H. P. Corwith, Ben Kievit, Jr., F. R. Lack, George Lewis, Knox McIlwain, G. D. O'Neill, B. J. Thompson, and H. P. Westman, secretary.

As chairman of the previous committee, Mr. Thompson indicated that, in his opinion, additional work might well be done in the field on cathode-ray tubes and high-frequency tubes.

It was felt wise to establish a general committee on graphical and literal symbols for standardization purposes. The possibility of the Technical Committee's sponsoring a symposium on electronics either during the Convention or at some other time was discussed and a subcommittee appointed to prepare specific recommendations.

Institute Meetings

Atlanta Section

A "student" meeting of the Atlanta Section was held on May 19 at the Atlanta Athletic Club. There were nineteen present and C. F. Daugherty, chairman, presided.

Three papers were presented by students of the Georgia School of Technology. The first by D. A. Howard was on "Notes on the Design of a Doherty Amplifier." He outlined first the limitations of class B radio-frequency amplifiers, presenting then the theory of the Doherty amplifier. He then described an amplifier of this type which was constructed at the college. The paper was concluded with a description of the design of an amplifier for amateur use giving constants and tuning-adjustment procedures.

The second paper on "Network Broadcasting" was by R. L. Adams who outlined a number of advantages of this service. Routes of program circuits serving the southeast were discussed. He next covered the transmission characteristics of wire circuits and the use of compensating networks. Data were given on transmission losses and the performance and spacing of repeater stations. The problem of keeping such equipment in operation was then considered.

The third paper on " Construction of a High-Frequency Induction Furnace" was presented by W. M. Furlow. The subject was introduced by a discussion of the uses of electric furnaces with particular reference to the heating of vacuum-tube elements for degassing during evacuation. A particular furnace constructed by the speaker was then described. Various experiments to determine optimum values of circuit constants, frequency, and load coupling, to obtain maximum heating were described. The paper was concluded with a description of a 5-kilowatt amplifier to be constructed for operation with this furnace.

It was the decision of the judges that Mr. Furlow be given Associate membership dues for one year as the prize for the best paper.

On June 9 the section met at the Atlanta Athletic Club and there were thirty-four present. Chairman Daugherty presided.

J. F. Morrison, radio engineer of Bell Telephone Laboratories, presented a paper on "Antennas and Transmission Lines." He pointed out that tower antennas for broadcast use are customarily judged on three factors; field intensity at one mile, radiation pattern, and impedance at the point of excitation. It was pointed out that towers approximately 0.55 wavelength high are considered satisfactory.

Theoretical concepts of radiation and the relation of currents in different parts of a tower were covered next. It was then pointed out that any coupling means which will permit an effective transfer of power from the transmitter to the antenna would be satisfactory. He then discussed the shunt-fed system in which the transmission line is connected across a sufficient length of the lower end of a grounded vertical radiator to obtain a satisfactory coupling impedance. Tuning

and impedance-matching procedures were then described. Typical performance characteristics of such a system were compared with insulated antennas.

The paper was closed with a discussion of transmission lines which included open-wire and coaxial systems. The performance of these lines and formulas giving various constants were discussed. It was pointed out that it is generally difficult to maintain sufficient balance in 2- and 4-wire open lines to minimize radiation.

Buffalo-Niagara Section

On May 18 the Buffalo-Niagara Section met at the University of Buffalo. G. C. Crom, Jr., Chairman, presided and there were forty-seven present.

"Various Methods and Measurements Required in Television Laboratory Work" was the subject of a paper by S. W. Seeley, RCA License Division Laboratories. It was pointed out that phase differences which were unimportant in oral transmission made television transmissions unintelligible. The light spot moves about two and one-half miles a second. While the wave is traveling along the antenna length, the spot lags by several hundred feet, causing reflections. Reflections are also caused by reradiation from earth formations and man-made structures.

Amplifiers are designed to handle direct-current signals. The time required for a signal to travel from the antenna binding post to the grid of the last tube may be equivalent to that of about five complete cycles. This is about one and one-half microseconds.

With low light intensity, thirty pictures per second produces no flicker although some is produced at high intensity. Motion pictures are projected forty-eight per second, twenty-four being pictures and twenty-four blanks. A tentative standard for television requires a transmission of sixty pictures per second with 441 lines per picture. A picture-signal generator was used to demonstrate some of the things discussed.

F. E. Terman, head of the electrical engineering department of Stanford University, presented a paper on "Detectors—Distortionless and Otherwise" at the June 8 meeting. Chairman Crom presided and the attendance was forty-seven. The meeting was held at the University of Buffalo.

A summary of detection theory was first presented. Fundamental diode circuits were then given and principals of operation explained. Such factors as distortion, efficiency, input impedance, and radio-fre-

quency voltage developed across the output circuit were covered. Making certain generalities, the diode efficiency of rectification was shown to be equal to the ratio of the alternating-current impedance to the direct-current resistance. The 6H6 was shown to have the highest efficiency of existing diodes reaching a value of ninety to ninety-five per cent. The diode does not have the same modulation capability on the envelope and side bands. Methods of overcoming this deficiency were noted. It was pointed out that diode detectors give low distortion, will not overload, have a high input impedance, and good frequency response. Apparatus for obtaining experimental data was described. It was pointed out that in the design of these circuits, the input and output circuits must be considered as a unit with the diode itself. Data have recently been published to permit reasonably accurate calculation of the characteristics of a given diode-circuit design.

This was the annual meeting of the section, and in the election of officers H. C. Tittle, chief radio engineer of the Colonial Radio Corporation, was elected chairman; F. J. Smith, inspector for the Federal Communications Commission, was designated vice chairman; and E. C. Waud was re-elected secretary-treasurer.

Chicago Section

On June 10, the Chicago Section met in the Stevens Hotel. J. E. Brown, chairman, presided and there were 200 present. This meeting was held in co-operation with the National Radio Parts Trade Show Convention.

L. C. F. Horle, consultant, presented a paper on "Notes on Cross Modulation." He described principally the important nature of the problem of interference caused by external cross modulation. The Radio Manufacturers Association's interest in the problem was outlined.

In the discussion, F. Fenton described some important sources of interference associated with utility systems and the equipment used by their customers. F. Johnston described some experiences met in surveys made by WLW when their high-power transmitter went into operation. J. K. Johnson described cross modulation in receivers and demonstrated its measurement by the two-signal test. One design system to avoid this trouble employs a controlled tube as an element in the antenna-coupling system. This tube affects the coupling between the antenna and first radio-frequency or converter tube and keeps signals within limits which avoid cross modulation.

Cleveland Section

Meetings of the Cleveland Section were held on May 26 and May 28 and presided over by L. N. Chatterton, chairman. The earlier meeting, which was attended by fifty-six, was an inspection of the new transmitter plant at WGAR and was conducted by R. M. Pierce technical supervisor of that station. He outlined first the problems of obtaining a license for increased power. The licensing authority required the erection of a directional antenna so as to avoid interference to listeners of stations located in Connecticut, New Jersey, District of Columbia, Georgia, and Louisiana.

Following this discussion, a general tour of the station was made and included a demonstration of the flexibility and capabilities of the new 5000-watt transmitter. In order to make these demonstrations, the program was switched over to the old transmitter.

The meeting on the 28th was held at the Hotel Statler and attended by twenty-one. "Radio Equipment in Air Transport Service" was the subject of a paper by Claude King of the Cleveland Airport. He described briefly the development of radio in the control of air transportation and covered radio beacons, weather-reporting service, and airport control. The paper was closed with a discussion of blind-landing systems. In the discussion of the paper, it was pointed out that developments in landing beams are progressing more rapidly than other methods of blind landing, the use of warning transmitters on antenna towers was discontinued because of interference with regular directive signals, landing-beam-transmission frequencies lie between 90 and 120 megacycles, visual landing indicators are in greater favor than aural devices and no correlation has been obtained between elevation and the pattern of snow static encountered.

A motion picture, "Coast to Coast By Air," was then projected after a brief talk on the general problems of air transportation by W. P. Feiten, traffic manager of the Cleveland district for United Air Lines.

Emporium Section

On June 14, the Emporium Section met in the American Legion Room. There were forty-six present and A. W. Keen, chairman, presided. Professor Terman presented his paper on "Diode Detectors—Distortionless and Otherwise" which has been summarized in the report of the June 8 meeting of the Buffalo-Niagara Section.

Philadelphia Section

On June 9, 220 attended a meeting of the Philadelphia Section held at the Engineers Club. A. F. Murray, chairman, presided.

Professor Terman presented his paper on detectors at this meeting and it was discussed by Messrs. Applegarth, Barton, Laport, and Pollack.

As this was the annual meeting, the election of officers was held and resulted in the designation of H. J. Schrader, RCA Manufacturing Company, as chairman; R. S. Hayes, Bell Telephone Company of Pennsylvania, was named vice chairman; and R. L. Snyder was re-elected as secretary-treasurer.

San Francisco Section

The June 6 meeting of the San Francisco Section was held in Manning's Coffee Cafe and was presided over by Carl Penther, vice chairman. There were thirty-two present. The paper on "Harmonic Generation," by H. J. Scott and L. J. Black which was published in the April, 1938, Proceedings, was reviewed under the direction of L. J. Black of the University of California.

"Grid-Current Flow as a Factor in the Design of Vacuum-Tube Power Amplifiers," by W. L. Everitt and Karl Spangenberg, which appeared in the May, 1938, Proceedings, was reviewed by Karl Spangenberg of Stanford University.

The June 22 meeting was held in the Pacific Telephone andTelegraph Company Auditorium. Vice-Chairman Penther presided and there were thirty-three present.

R. L. Sink of Stanford University presented a paper on "Geometrical Electron Optics." He treated electrostatic focusing generally, pointing out similarities between light optics and electron optics. The requirements for such focusing were outlined and the derivation of formulas for computing electron trajectories was given.

Seattle Section

The May 27 and June 9 meetings of the Seattle Section were held at the University of Washington and presided over by R. O. Bach, vice chairman.

At the May 27 meeting which was attended by fifty, a paper on "A New Use for Radio in the Field of Fire Prevention and Criminal Apprehension," was presented by W. L. Foss, consulting engineer for the Houghton Radio Alarm Company. He described a device which, actuated by conventional systems for detecting burglary or fire, broad-

casts an alarm on the local radio-police channel giving by means of recorded speech the nature and location of the emergency. The operation of unattended radio transmitters was discussed and means described of preventing a delay in placing a transmitter in operation if the channel is already in use. Experimental models of reproducing equipment employing disk records were also demonstrated as well as a method of recording magnetically on a steel wire.

The June 9 meeting was attended by forty-two and was devoted to the presentation of three papers by students of the University of Washington. First, "A New Beam-Type Vacuum Tube," by Milton Pierce described the construction and testing of a new design of tube in which a sharply focused beam of electrons is projected from the cathode to an anode target. Electrostatic deflection plates control this beam moving it laterally on and off the target. By choosing a suitable operating position for the beam and shape for the anode, a variety of conductance curves are obtained.

The second paper on "Relative Penetration of 10- and 80-Meter Waves into Reinforced-Concrete Buildings" was by William Harrold and Myron Swarm. It described tests which have been in progress for two years to determine the optimum frequency for a campus police-radio system. Complete data are not yet available but the results of numerous observations were presented.

The third paper on "Auditory Perspective on a Single Carrier" was by Neil Sandstedt and covered a continuation of the work on simultaneously impressing amplitude and frequency modulation on a single carrier which was described to the Section in 1935 by J. R. Woodyard. It permits the transmission of two channels on a given carrier and requires the use of additional equipment with the standard amplitude-modulated transmitters and receivers. Cross talk between the two channels is adequately low for this purpose. Auditory perspective was demonstrated.

The judges awarded the first prize of $10.00 to Milton Pierce and the second prize of a year's Student membership in the Institute to the authors of the second paper, William Harrold and Myron Swarm.

Washington Section

E. H. Rietzke, chairman, presided at the June 13 meeting of the Washington Section which was held in the Potomac Electric Power Company Auditorium and attended by 130.

A paper on "Ultra-High-Frequency Technique" was presented by P. D. Zottu of the RCA Manufacturing Company. Measurements,

circuit elements, oscillators, receivers, and the general technique of handling superfrequencies were thoroughly discussed. Particular attention was directed to the use of distributed-constant circuits at these frequencies and the construction of various units were described and illustrated.

A "paper" by E. K. Jett and G. G. Gross of the Federal Communications Commission on the "Recent Cairo Radio Conference" was presented in the form of informal remarks on the important actions taken at the conference. Mr. Gross covered in general the regulatory side and Mr. Jett that dealing with the allocation of waves.

Proceedings of the Institute of Radio Engineers
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TECHNICAL PAPERS

A HIGH-EFFICIENCY GRID-MODULATED AMPLIFIER*

By

F. E. Terman and John R. Woodyard
(Stanford University, California)

Summary—*A method of applying impedance inversion to the grid-modulated amplifier is described. Two tubes are employed in the modulated stage with their plate circuits connected through the electrical equivalent of a quarter-wave transmission line, and the modulating voltage is applied to the grids in series with the grid bias. By this means, high efficiency and large power output per tube are obtained. Efficiences of 65 to 80 per cent are readily secured during both modulated and unmodulated intervals as compared with 60 to 65 per cent for the high-efficiency linear amplifier. The output per tube is also greater than that of the high-efficiency linear amplifier. Only a small amount of modulating power is required, and this system is particularly adaptable to negative feedback.*

The effect of the impedance-inverting network on modulated waves is discussed. A simplified method of adjusting the tuned circuits of the impedance inverter is described.

Formulas are derived for output and efficiency for both the high-efficiency grid modulator and the high-efficiency linear amplifier.

Introduction

THERE are two general methods of applying amplitude modulation to a radio transmitter. The radio-frequency current may be modulated either before or after it reaches the final amplifier tube, resulting in low- and high-level modulation, respectively. In low-level modulation the amount of audio-frequency power required is small, which is an advantage, but the modulated wave must be amplified by a linear, or class B, amplifier which has a relatively low efficiency. In present systems of high-level plate modulation, on the other hand, high-efficiency class C amplification is used for the radio-frequency currents, but a large amount of audio-frequency modulating power is required. Since this audio-frequency power must be supplied by an amplifier which has a low efficiency, the over-all efficiency is still relatively low.

As efficiency and power output per tube have become of greater importance with the continually increasing power of broadcast stations, several systems have been proposed for overcoming these

* Decimal classification: R355.7. Original manuscript received by the Institute, February 24, 1938. Presented before Pacific Coast Meeting, Spokane, Washington, September 2, 1937.

limitations. One of these, termed outphasing modulation[1] takes advantage of the fact that though a wave of variable amplitude must be amplified by an inefficient linear amplifier, a wave of constant amplitude can be amplified by a highly efficient nonlinear class C amplifier. In the outphasing system the amplitude-modulated wave is changed to a phase-modulated wave before amplification, and is then changed back into amplitude modulation after amplification, using a phase-shifting method. In another method of obtaining increased efficiency, an attempt is made to vary certain plate or grid voltages in the transmitter in proportion to the average amplitude (or envelope) of the sound wave. These variations do not take place instantly, however, so that such arrangements introduce distortion and cannot be used where high-quality transmission is desired. The controlled-carrier modulator, the dynamic-shift linear amplifier, and the dynamic-shift grid-modulated amplifier, are examples of this method.[2,3,4] Another method of attacking the problem, and the most successful to date, is the high-efficiency linear amplifier described by Doherty[5] in which the load impedance, rather than the operating voltages, of a linear amplifier is shifted to take care of the peaks of the modulation cycle.

High-Efficiency Grid Modulation

This system makes use of the impedance-inverting property of a quarter-wave transmission line such as was used by Doherty to increase the efficiency of a linear amplifier. However, in the present method, modulation occurs in the final radio-frequency amplifier tubes rather than in a low-level stage. In previous systems of grid modulation the carrier output per tube has tended to be low because it was necessary to adjust the load impedance to the proper value for conditions at the peak of the modulation cycle, so that at average or carrier level the load resistance was too low for efficient operation. This limitation can be overcome by making use of a quarter-wave line connected as shown in Fig. 1. It can easily be proved that at the frequency for which L and C are resonant, $R_s = Z_0^2/R_R$, where R_s is the apparent resistance offered at the sending end of the line across the terminals AA' when a resistance of R_R ohms is connected at the receiving-end terminals BB',

[1] H. Chireix, "High power outphasing modulation," Proc. I.R.E., vol. 23, pp. 1370–1392; November, (1935).

[2] F. E. Terman, "Radio Engineering," second edition, pp. 536–539, McGraw-Hill Book Company, (1937).

[3] F. E. Terman and F. A. Everest, "Dynamic-shift grid-bias modulation," *Radio*, no. 211, p. 22; July, (1936).

[4] J. N. A. Hawkins, "A new high efficiency linear amplifier," *Radio*, no. 209, p. 9; May, (1936).

[5] W. H. Doherty, "A new high efficiency power amplifier for modulated waves," Proc. I.R.E., vol. 24, pp. 1163–1182; September, (1936).

and Z_0 is the characteristic impedance of the line. That is, the sending-end resistance is inversely proportional to the receiving-end resistance, so that the line functions as an impedance inverter.

Consider first what would happen if tube No. 1 were working alone as an ordinary grid modulator with the quarter-wave line removed and the load placed directly in its plate circuit. Assuming 100 per cent modulation, if the load impedance were correctly adjusted to give low minimum plate voltage and high efficiency at modulation peaks, then a quarter of a modulation cycle later, when the audio-frequency signal voltage E_{s1} is passing through zero, the minimum plate voltage would

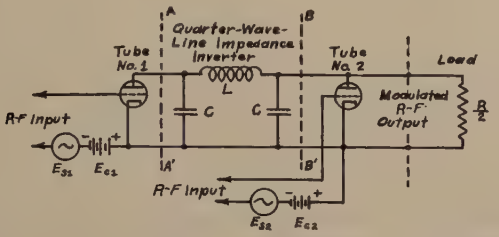

Fig. 1—Simplified circuit diagram of high-efficiency grid-modulated amplifier.

be slightly more than half the direct plate voltage, and the efficiency would be reduced to one half of its peak value. This low efficiency at carrier level could be overcome by doubling the load resistance so that the minimum plate voltage would be low, and consequently the efficiency high, under carrier conditions. However, since the alternating plate voltage cannot ordinarily be greater than the direct plate voltage, the load voltage would then be unable to increase during modulation peaks, and distortion would result.

Evidently, a variable load impedance is needed which increases as the modulation cycle goes from peak to carrier level, so that the alternating voltage between the plate and the cathode remains constant and the efficiency high. Such a nonlinear impedance is supplied by the combination of quarter-wave line and tube No. 2 shown in Fig. 1. A quarter-wave line, neglecting losses, has the property of transforming power at constant voltage and variable current into power at constant current and variable voltage.[6] Since it is desired to have the voltage delivered to the antenna vary while, in tube No. 1, the voltage remains constant and the current varies, the quarter-wave line is a useful connecting link between the two. Tube No. 2, which is biased so that it

[6] This property was discovered by Steinmetz and described by him under the name "monocyclic" circuit. C. P. Steinmetz, "Theory and Calculation of Electric Circuits," pp. 260–261, McGraw-Hill Book Company, New York, (1917).

does not begin to pass plate current until carrier level is reached, serves the double purpose of furnishing a variable impedance and supplying additional power during the positive peaks of the modulation cycle.

Before the operation of Fig. 1 can be explained in detail it will be necessary to consider the impedance relations of the various parts of the circuit. The first step in applying impedance inversion to grid modulation is to calculate[7] or otherwise determine the proper load impedance R which tube No. 1 should work into as a class C amplifier at peak level in order to deliver maximum possible power output consistent with allowable plate dissipation, available cathode emission, and allowable direct plate voltage. This value of plate dissipation can be made higher than for unmodulated class C operation because the tube is operating at peak level only a small fraction of the time. Also, the allowable direct plate voltage can be made higher than for a plate-modulated class C amplifier because the instantaneous plate-supply voltage does not rise during modulation and put additional voltage strains on tube and apparatus, as it does in a plate-modulated amplifier.

Having determined this value of load impedance, the actual load placed in parallel with tube No. 2 is made equal to $R/2$. The characteristic impedance of the quarter-wave line is made equal to R; that is, the reactance of inductance L at the carrier frequency is R, and the reactance of each of the condensers C is also equal to R. Signal, (i.e., modulating) voltages E_{s1} and E_{s2} are applied to the two tubes in the same phase.

The operation may now be explained as follows. The bias on tube No. 2 is adjusted so that between zero and carrier level this tube does not pass current. The load impedance on No. 1 is then the reciprocal of the actual load impedance times the square of the characteristic impedance, or $2R$. This high load impedance enables No. 1 to work at twice the efficiency which it otherwise would have for levels not exceeding the carrier. It will be noted that this causes the efficiency at carrier level to be the same as in a normal class C amplifier, instead of only half as great as in an ordinary grid-modulated amplifier.

When carrier level is reached, No. 2 starts to pass plate current in the form of short pulses, which increase in magnitude until, at peak level, this tube is also delivering its full rated output to the load. Since No. 2 delivers power to the load, it presents an apparent negative resistance across the output of the impedance inverter in parallel with the actual load. This causes the apparent load resistance presented to the output terminals of the impedance inverter to increase as the

[7] F. E. Terman and W. C. Roake, "Calculation and design of class C amplifiers" Proc. I.R.E., vol. 24, pp. 620–632; April, (1936).

modulation cycle goes from carrier to peak level, which means that the input resistance of the inverter decreases. Tube No. 1 is thus able to supply a constantly increasing amount of power without increasing its radio-frequency plate voltage (which was already at its maximum possible value) as the modulation goes from carrier to peak level.

It will be noted that tube No. 1 operates as a class C amplifier with a relatively large alternating voltage between the plate and the cathode most of the time, and so has high average plate efficiency. At the same time No. 2 also operates as a class C amplifier with high efficiency, so the over-all average efficiency is high.

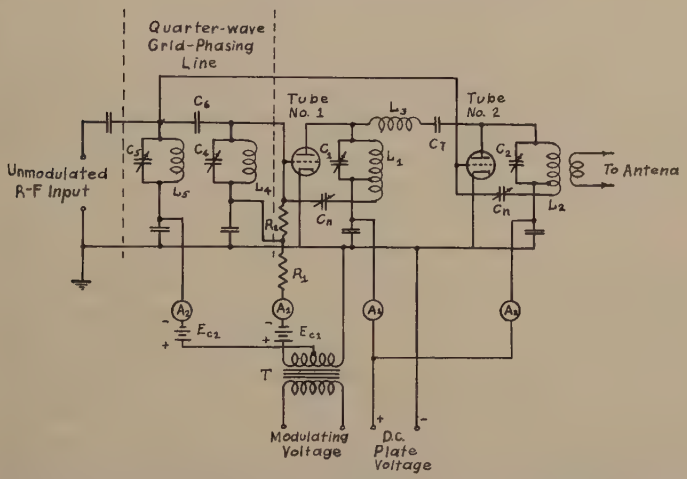

Fig. 2—Practical circuit for high-efficiency grid-modulated amplifier.

Practical Circuits

A practical circuit for placing these principles in operation is shown in Fig. 2. Here the 90-degree phase shift produced by the quarter-wave line in the plate circuit is compensated for by an equal and opposite phase shift produced by a quarter-wave line of opposite type in the grid circuit of Fig. 2. Furthermore, the shunting reactances of the lines are supplied by parallel resonant circuits detuned to give the required reactance.

The bias voltages applied to the two tubes are different, since tube No. 2 must not operate until carrier level is reached. In addition, tube No. 1 is also provided with a grid-leak bias resistor R_1 to prevent the grid of this tube from going excessively positive at the modulation peaks when the operating conditions are such that this grid goes moderately positive at carrier level.

The modulating voltage is applied to the two grids in the same phase, but with different magnitudes, by means of the tapped transformer T. With ordinary triodes tube No. 2 will require from one half to two thirds as much modulating voltage as No. 1. The exact ratio can be determined experimentally as the condition giving the best linearity of modulation.

The radio-frequency voltages applied to the grids of the two tubes can be the same or different according to the adjustment of the resistance R_2 terminating the line in the grid circuit of tube No. 1. This resistance may or may not be required, depending on how much it is desired to let the grids go positive, and the characteristic impedance of the grid line.

Of the remaining circuit elements C_6 is the series element of the grid line, and its reactance is made equal to the desired characteristic impedance of the line. C_7 is a blocking condenser that makes it possible to read the plate current of each tube separately. L_3 is the same inductance shown as L in Fig. 1.

Adjustment of Circuits

The grid line of tube No. 1 must be adjusted experimentally to have a phase shift of exactly 90 degrees. In theory, a cathode-ray oscillograph could be used by connecting one pair of plates across each end of the network and adjusting for a circle on the screen, but this method is generally unsatisfactory as harmonics in the voltages cause the accuracy of setting to be very poor, and the capacitance of the oscillograph disturbs conditions. A better method, which does not require the connection of any apparatus to the radio-frequency circuits, is to couple the radio-frequency input loosely to the driver, after having fixed C_6 at the desired value as explained above, and then vary C_4 while watching the plate-current meter of the driver tube. A maximum will be passed through at series resonance between C_6 and L_4C_4, which is the correct adjustment. Before the final adjustment of C_4 is made the neutralizing condensers should be adjusted, which can be done by any of the conventional methods. C_5L_5 should also be adjusted to resonance, but is not critical as it does not affect the relative phases of the grid voltages.

The radio-frequency excitation voltages on the grids of the two tubes are preferably adjusted to the calculated value[7] without connecting any measuring device, such as a vacuum-tube voltmeter, which might have sufficient capacitance to change the tuning and disturb the voltages being measured. This can be accomplished by using the power tubes as vacuum-tube voltmeters. To do this the direct-

voltage supply to the plate is disconnected, the radio-frequency voltage is applied to the inputs, and the negative grid bias on each tube increased until grid current just ceases to flow. The peak radio-frequency voltages are then equal to the bias potentials.

The most difficult part of the circuit to adjust is the impedance-inverting network in the plate circuits. This applies to the high-efficiency linear amplifier as well as the grid modulator. One reason for this difficulty is that there are two separate conditions which must be fulfilled. The first is that impedances must be accurately inverted (i.e., the line must be a quarter-wave line), while the second condition is that the characteristic impedance must also be correct.

When the end sections are properly adjusted, the characteristic impedance is determined by the reactance of L_3. However, the equivalent series reactance of L_3 will not be the value computed from the inductance, but will be somewhat greater because of partial resonance with the distributed capacitance of the coil and wiring. Some method is hence needed for experimentally determining the proper value of L_3 under actual conditions. A cathode-ray tube could theoretically be used, but has the disadvantage of introducing capacitance. Small harmonic voltages across the tank circuits also cause trouble when a cathode-ray tube is used. A simpler and more satisfactory technique is as follows.

A load resistance of the proper value is first connected directly across the output L_2C_2. If the value previously computed as correct for each tube at peak level is denoted by R, the amount of resistance used is $R/2$. The filament of tube No. 2 is then opened, but this tube is left in the socket so that its capacitance will be present. Since the tube has been neutralized, the tube capacitances will be the same with the filament cold as when it is operating. The normal value of radio-frequency grid excitation is applied, and the direct grid voltage of tube No. 1 set at any convenient value. Plate voltage is applied, preferably of reduced value or through a protective resistance. Condenser C_1 is then adjusted for minimum plate current of No. 1 as in any class C amplifier, while C_2 is adjusted for maximum plate current. It will be found that the setting of C_1 for minimum plate current depends on the setting of C_2, and that the setting of C_2 for maximum plate current depends on the setting of C_1, so that the adjustments are not independent of each other. It is shown in Appendix II that the correct condition for quarter-wave operation of the plate line will be obtained if the adjustments are made so as to give the highest minimum plate current or lowest maximum plate current. This may be explained in more detail as follows. If C_2 is left fixed temporarily and C_1 is varied, a

minimum will be passed through. If C_2 is then shifted a small amount and C_1 again varied, a minimum is again passed. This second minimum will, in general, be at a different capacitance on C_1 and will be a different value of direct plate current. C_2 may be changed in steps of one or two divisions while varying C_1 slowly back and forth until the particular combination giving the highest possible minimum is found, which is the correct combination to use. In carrying out this procedure it is essential that the minimum for each setting of C_2 be obtained and recorded. It is not possible to adjust C_1 and C_2 alternately to get the highest minimum as the adjustments then diverge away from the correct value.

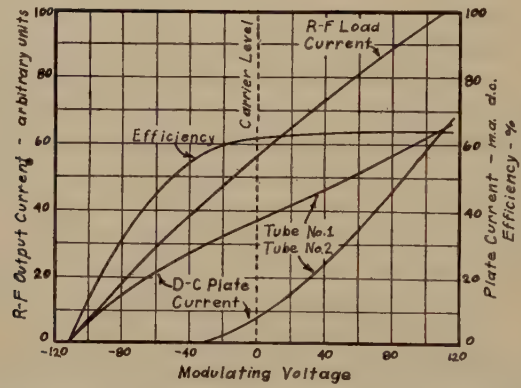

Fig. 3—Experimental characteristics of small high-efficiency grid modulator.

As an alternative method, C_1 can be changed in steps and C_2 continuously varied, when a series of maxima will be observed. The correct tuning in this case is the value of C_1 which gives the lowest possible maximum as C_2 is varied. Both ways will arrive at the same values of capacitance.

The above tuning procedure insures that the impedance inverter is working properly, but it tells us nothing about whether or not its characteristic impedance is correct. The characteristic impedance can, however, be determined by shifting the load from one end of the inverter to the other as follows. With the filament of tube No. 2 opened and of No. 1 closed, and a load of $R/2$ across No. 2, the condensers C_1 and C_2 are tuned as explained above and the plate current is noted. Then the series element L_3 is opened and a resistance $2R$ is placed directly across L_1C_1, which is retuned for minimum plate current. The minimum plate current should then have the same value as before. If it does not the inductance of L_3 was wrong. L_3 may then be changed

slightly, and the process repeated until $R/2$ across the output of the inverter draws the same plate current as $2R$ across the tube directly.

As a final check on the operating conditions, $R/2$ is replaced across the output, both filaments are closed, and the calculated values of radio-frequency grid excitation, direct plate voltage, and grid bias are applied. At the positive peak of the modulation voltage both tubes should then have the same direct plate current and radio-frequency plate voltage and should deliver the same power. At carrier level, tube No. 1 should be passing approximately one half as much plate current as at peak level, and No. 2 should be just starting to pass plate current, as shown in Fig. 3. At the trough of modulation, tube No. 1 should be just starting to pass plate current. Transformer T in Fig. 2 may be replaced by a tapped resistor and variable direct voltage for the purpose of making these adjustments. If the plate currents are not equal at positive modulation peaks, the grid leak R_1 may be changed, or the ratio of radio-frequency grid voltages can be changed slightly by adjusting R_2. Then, as modulating voltage is applied, the radio-frequency load current should change linearly. Finally $R/2$ is replaced by the antenna, by coupling the antenna to the tank circuit L_2C_2.

Experimental Results

In order to check the theory, a small unit was constructed using 2A3 tubes and the circuit of Fig. 2. The transformer T was replaced by a tapped resistor, with a source of variable direct voltage in place of the modulating voltage. Results obtained from point-by-point measurements are shown in Fig. 3. It will be observed that the results are approximately as predicted by theory. The plate current of tube No. 1 increases almost uniformly. No. 2 starts to conduct approximately at carrier level and increases twice as fast as No. 1 until the two curves meet at peak level. Efficiency increases rapidly up to about carrier level and then remains approximately constant, which means that the average efficiency during modulation will be high. The radio-frequency output-current characteristic is approximately a straight line and could probably be made still more distortionless by further refinements of adjustment. The purpose of this investigation was not to see how little distortion could be obtained, but rather to show the correctness of the principles, and that the system can be made to work. The values of efficiency shown in Fig. 3 should not be taken as what can be obtained in practice, as these data were taken with low direct plate voltage. With ordinary transmitter tubes efficiencies considerably higher could readily be obtained.

General Considerations

In the above discussion of the effect of the so-called quarter-wave lines in the plate and grid circuits, it was assumed that these networks actually acted as quarter-wave lines at the frequency applied to them. This property, in the plate-circuit network for example, depends upon the reactances of the series and shunt elements of the line. Since the reactance of inductances and parallel-tuned circuits varies with frequency, the correct conditions for impedance inversion cannot be exactly fulfilled at the higher side-band frequencies. This applies to the high-efficiency linear amplifier as well, and is somewhat analogous to

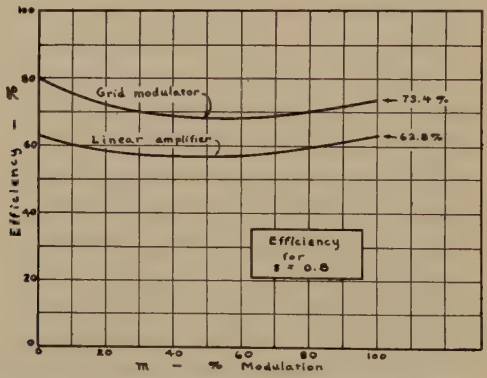

Fig. 4—Over-all efficiency as a function of m for $s = 0.8$, where s is the efficiency factor of each tube at modulation peaks.

an ordinary class C amplifier in which the tank circuit is not quite correctly tuned for the side bands of high modulation frequencies. In the case of high-quality systems it is hence worth considering the possibility of a network which would act as an impedance inverter over a wider frequency band. In some cases, it might be desirable to construct a network which is more nearly the true electrical equivalent of a transmission line, as for example by using two or three smaller sections connected end to end instead of one large section.

Calculations of efficiency, plate loss, and power output per tube are given in Appendix I, with results shown in the curves of Figs. 4, 5, and 6. Corresponding calculations for the Doherty high-efficiency linear amplifier system are also shown for comparison. It will be observed that the grid-modulated system is not limited to a theoretical maximum efficiency of 78.5 per cent in tube No. 1, as is the high-efficiency linear amplifier. As a result, over-all efficiencies somewhat higher, as well as larger outputs per tube, are obtained with the high-efficiency grid modulator. Fig. 5 shows that the plate loss in tube No. 1

is much smaller for the grid modulator. Furthermore, the plate loss in tube No. 2 is less than in No. 1, so that a tube with smaller dissipation

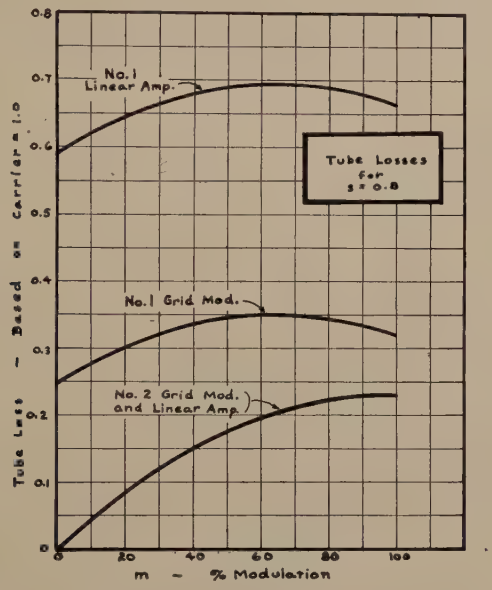

Fig. 5—Individual tube losses as a function of m.

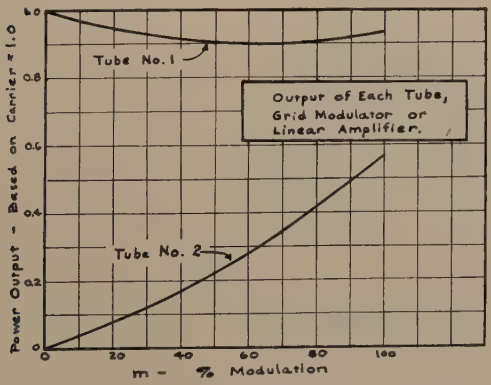

Fig. 6—Output of each tube as a function of per cent modulation. These curves apply to both the grid modulator and the linear amplifier.

rating (or fewer tubes) could be used in No. 2 position when highest possible tube economy is desired.

The application of negative feedback is particularly simple in the grid modulator. This is true because, being a high-level modulation system, only one radio-frequency stage is involved, so that difficulties

with phase shift are greatly reduced. Furthermore, the amount of modulating power required is small so that only a small fraction of the radio-frequency output power needs to be rectified to operate the feedback circuit. The advantages of negative feedback in a transmitter, in eliminating amplitude distortion and hum due to alternating filament current are well known.

High-efficiency grid modulation can, of course, also be applied to suppressor-grid or screen-grid modulators by placing the modulating voltage in series with the desired electrode.

Conclusions

It appears from the work which has been done that grid modulation when used in connection with impedance inversion has definite possibilities for high average efficiency and high output per tube. Since this is a high-level system of modulation, it possesses the well-known advantages of high-level modulation, but in addition has the advantage of low-level modulation in that it requires only a very small amount of modulating power. Calculations indicate, for example, that two tubes rated at 10-kilowatt carrier each when operated class C with high-level plate modulation, would be capable of the same total output, or a 20-kilowatt carrier with 80-kilowatt peaks, when used in the high-efficiency grid modulator. Furthermore three tubes would be capable of approximately twice this output if the filament emission were sufficient. The plate voltage does not increase during positive modulation peaks, as it does with plate modulation. Therefore the direct plate-supply voltage can be safely increased, and since it is cheaper to supply power in the form of high-voltage direct current than in the form of audio frequency, one advantage over plate modulation is obvious. The total power consumption of a transmitting station employing high-efficiency grid modulation is smaller than even that of a station using high-efficiency linear amplification. Negative feedback is particularly easy to apply to the proposed system because fewer radio-frequency stages are involved than in low-level systems, such as systems using the high-efficiency linear amplifier.

Appendix I

Calculation of Performance

Fig. 7(a) shows the shape of the instantaneous-efficiency curves for an ideal high-efficiency grid modulator. It is based on

$$\text{efficiency} = \frac{E_B - E_m}{E_B} \tag{1}$$

where E_B = direct plate-supply voltage, and E_m = minimum instantaneous plate voltage. This assumes that the plate current flows in short pulses at the time of minimum plate voltage, and is reasonably accurate for ordinary conditions of class C operation. The independent variable k in Fig. 7 is a quantity which is proportional to the instantaneous

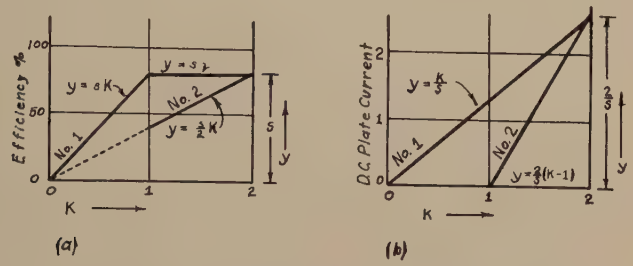

Fig. 7—Instantaneous characteristics of an ideal high-efficiency grid-modulated amplifier.

radio-frequency envelope amplitude, with $k=1$ being carrier level. The variable ordinate is denoted by y. The parameter s is defined by the relation

$$s = \frac{E_B - E_m'}{E_B} \tag{2}$$

where E_m' is the minimum instantaneous plate voltage at positive peaks of the modulation cycle. Therefore s is the efficiency obtained at modulation peaks, and depends on circuit constants and operating voltages. Under usual conditions s will lie between 65 and 85 per cent.

Fig. 7(b) gives the direct plate currents of the individual tubes plotted in arbitrary units as a function of k. These curves can be derived from Fig. 4(a) and the inverting properties of the impedance inverter. For purposes of calculation, the direct plate voltage and carrier power are taken as unity.

By making use of Fig. 7, the average efficiency as a function of per cent of modulation can be calculated as follows. Let

$$k = 1 - m \cos pt \tag{3}$$

where m = degree of modulation, $p = 2\pi F$, and F = audio frequency. The average input to either tube will then be the quantity $\int i dt$ averaged over an audio-frequency cycle.

$$\text{No. 1 input} = \frac{1}{\pi}\int_0^{\pi} \frac{k}{s}\,dx = \frac{1}{\pi s}\int_0^{\pi} (1 - m\cos x)dx = \frac{1}{s} \tag{4}$$

where $x = pt$.

$$\text{No. 2 input} = \frac{1}{\pi s}\int_{\pi/2}^{\pi} 2(1 - m\cos x - 1)dx = \frac{2m}{\pi s}. \tag{5}$$

Since the output equals the input times the efficiency,

$$\text{No. 1 output} = \frac{1}{\pi}\left[\int_0^{\pi/2}(1 - m\cos x)^2 dx + \int_{\pi/2}^{\pi}(1 - m\cos x)dx\right]$$

$$= 1 - \frac{m}{\pi} + \frac{m^2}{4}. \tag{6}$$

$$\text{No. 2 output} = \frac{1}{\pi}\int_{\pi/2}^{\pi}[(1 - m\cos x)^2 - (1 - \mathrm{m}\cos x)]dx$$

$$= \frac{m}{\pi} + \frac{m^2}{4}. \tag{7}$$

Adding (6) and (7), likewise (4) and (5), we have,

$$\text{total output} = 1 + \frac{m^2}{2} \tag{8}$$

$$\text{total input} = \frac{1}{s}\left(1 + \frac{2m}{\pi}\right) \tag{9}$$

$$\text{average efficiency} = s\,\frac{1 + \dfrac{m^2}{2}}{1 + \dfrac{2m}{\pi}}. \tag{10}$$

Subtracting the output from the input for the individual tubes gives the plate dissipation of each tube, based on a carrier output of one, to be the following:

$$\text{No. 1 plate dissipation} = \frac{1 - s}{s} + \frac{m}{\pi} - \frac{m^2}{4} \tag{11}$$

$$\text{No. 2 plate dissipation} = \frac{2}{\pi}\,\frac{1 - s}{s}\,m + \frac{m}{\pi} - \frac{m^2}{4}. \tag{12}$$

In the case of the high-efficiency linear amplifier, the same procedure applies, except for obvious modifications made necessary by the fact that the plate current in tube No. 1 flows in half cycles instead of short pulses so that this tube has a maximum possible efficiency of $\pi/4$. Making the same simplifying assumptions as before, the instan-

taneous-efficiency and plate-current curves will therefore be as shown in Fig. 8, as functions[8] of k. Integrating as before to find the average input and output of each tube gives the following:

$$\text{No. 1 input} = \frac{1}{\pi s}\int_0^{\pi}\frac{4}{\pi}(1 - m\cos x)dx = \frac{4}{\pi s} \tag{13}$$

$$\text{No. 2 input} = \frac{2m}{\pi s} \tag{14}$$

$$\text{No. 1 output} = 1 - \frac{m}{\pi} + \frac{m^2}{4} \tag{15}$$

$$\text{No. 2 output} = \frac{m}{\pi} + \frac{m^2}{4} \tag{16}$$

$$\text{total output} = 1 + \frac{m^2}{2}. \tag{17}$$

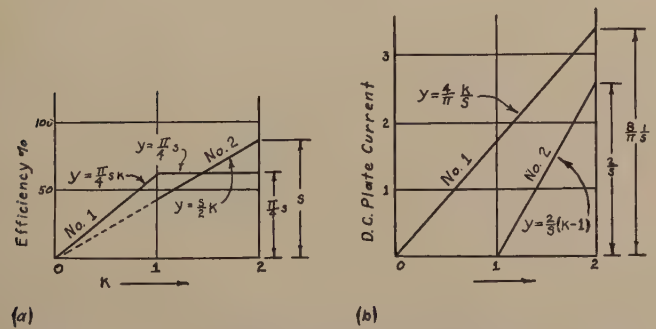

Fig. 8—Instantaneous characteristics of an ideal high-efficiency linear amplifier.

Equations (14), (15), (16), and (17) are the same as for the grid modulator. Adding (13) and (14) we have

$$\text{total input} = \frac{1}{s}\left(\frac{4}{\pi} + \frac{2m}{\pi}\right) \tag{18}$$

$$\text{average efficiency} = s\,\frac{1 + \frac{m^2}{2}}{\frac{4}{\pi} + \frac{2m}{\pi}} = s\,\frac{\pi}{4}\,\frac{2 + m^2}{2 + m}. \tag{19}$$

[8] Footnote added in proof: It will be observed that in the high-efficiency linear amplifier the two tubes should not be expected to carry equal direct plate currents at the modulation peaks because tube No. 1 operates at lower efficiency than tube No. 2. The curves of operation shown in the Doherty paper do have equal direct plate currents at the crest of modulation and so do not correspond to normal operation for the high-efficiency amplifier.

Subtracting (15) from (13) and (16) from (14) gives

$$\text{No. 1 plate dissipation} = \frac{4}{\pi s} - 1 + \frac{m}{\pi} - \frac{m^2}{4} \tag{20}$$

$$\text{No. 2 plate dissipation} = \frac{2}{\pi}\,\frac{1-s}{s}\,m + \frac{m}{\pi} - \frac{m^2}{4}. \tag{21}$$

The results of these calculations, for a typical case where $s=0.8$, are shown in Figs. 4, 5, and 6 as a function of the per cent of modulation m for both the grid modulator and the linear amplifier.

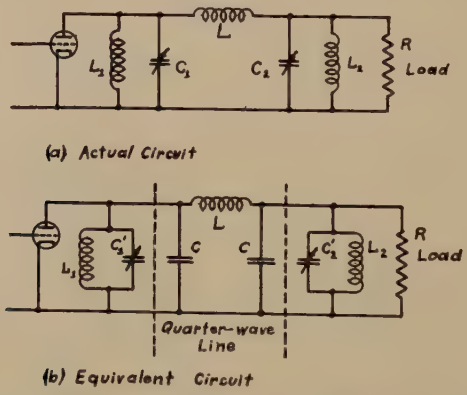

Fig. 9—Actual and equivalent circuits for analyzing an impedance inverter.

Appendix II

Analysis of Impedance-Inverter Adjustment

The purpose of this section is to justify mathematically the method of tuning the impedance inverter previously described. During the tuning, tube No. 2 is made nonconducting; therefore the actual network applied to the plate circuit of tube No. 1 may be represented as in Fig. 9(a), assuming that there are no losses in the coils or condensers. This circuit may be reduced to the equivalent circuit of Fig. 9(b) in which C_1' and C_2' are less than C_1 and C_2 by the amount C required to make up the quarter-wave line. The correct operating condition will then exist when

$$\omega L_1 = \frac{1}{\omega C_1'} \quad \text{and} \quad \omega L_2 = \frac{1}{\omega C_2'} \tag{22}$$

where ω is the angular velocity of the applied frequency.

Let the following notation be used in describing the operation of Fig. 9:

$Y = g + jb$ = input admittance which the combination of tuned circuits, quarter-wave line, and useful load presents to the plate of the tube

g = input conductance

b = input susceptance

$Y_s = g_s + jb_s$ = sending-end admittance of the quarter-wave line with load connected, and g_s and b_s are the corresponding conductance and susceptance, respectively. Then

$$Y = Y_s + jb_1 = g_s + j(b_1 + b_s) \tag{23}$$

where $b_1 = \omega C_1' - (1/\omega L_1)$. By elementary transmission-line theory we have

$$Y_s = \frac{Y_0^2}{Y_r} = \frac{Y_0^2}{g_r + jb_2} = Y_0^2\left(\frac{g_r}{g_r^2 + b_2^2} - j\frac{b_2}{g_r^2 + b_2^2}\right) \tag{24}$$

where Y_0 is the characteristic admittance of the line (i.e., the reciprocal of the characteristic impedance), $g_r = 1/R$ where R is the load resistance, and $b_2 = \omega C_2' - (1/\omega L_2)$. Combining (23) and (24) gives

$$Y = Y_0^2 \frac{g_r}{g_r^2 + b_2^2} + j\left(b_1 - Y_0^2 \frac{b_2}{g_r^2 + b_2^2}\right). \tag{25}$$

Suppose that C_2' is first set at some fixed value as specified in the tuning procedure previously described, and C_1' is then tuned for minimum plate current, that is, for minimum Y. This makes

$$b_1 = Y_0^2 \frac{b_2}{g_r^2 + b_2^2} \tag{26}$$

and (25) becomes

$$Y = Y_0^2 \frac{g_r}{g_r^2 + b_2^2} \tag{27}$$

That value of C_2' is then selected which makes Y in (27) a maximum, as also specified in the tuning procedure. This makes $b_2 = 0$. Placing $b_2 = 0$ in (26) gives $b_1 = 0$. We then have

$$b_1 = \omega C_1' - \frac{1}{\omega L_1} = 0 \qquad \text{and} \qquad b_2 = \omega C_2' - \frac{1}{\omega L_2} = 0 \tag{28}$$

which are the desired conditions given in (22).

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A UNIQUE METHOD OF MODULATION FOR HIGH-FIDELITY TELEVISION TRANSMITTERS*

By

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Summary—*Present-day high-fidelity 441-line television demands modulation frequencies as high as 4 megacycles. Tube capacitance and the flywheel effect of resonant circuits make such modulation difficult and inefficient when conventional methods are used.*

The author describes a system called "transmission-line modulation" in which modulation is effected between the radio-frequency generator and the antenna by means of a variable impedance connected across the radio-frequency transmission line. This impedance, consisting of a quarter-wave line terminating in the modulator tubes, is controlled by the voltage applied to the grids of these tubes.

At high video frequencies the plate efficiency and degree of modulation compare favorably with the conventional systems employed in sound broadcasting.

A 1-kilowatt experimental television transmitter employing this system, which may be modulated 80 per cent at frequencies up to 5 megacycles, is described. For demonstration purposes a 200-megacycle oscillator, modulated at frequencies up to 20 megacycles, is shown.

IN ORDER to provide a high-fidelity television signal for the field testing of receivers in Philadelphia, it became desirable to remodel Philco's experimental television transmitter. The requirements were these: An average field strength of 1 millivolt at a distance of 7 miles; the carrier frequency should be about 50 megacycles; the height of the antenna 210 feet; and the over-all frequency response substantially flat from zero to 4 megacycles. If the frequency-response curve could be made to rise at the higher frequencies so much the better.

The fidelity requirement was the outstandingly difficult portion of the assignment, yet it was the most important. With a modern 441-line system a distinct improvement in picture quality can be shown when the upper modulating frequency limit of the system is extended to 4 megacycles and beyond. A side-by-side comparison of such a system with one cutting off at 2.4 megacycles is very convincing.

Conventional Modulation Methods

It soon became evident that the modulator might be the "bottleneck" as far as picture fidelity was concerned. The amplifiers from the

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television camera to the modulator had been designed and built to pass the required video-frequency band. Naturally the conventional methods of modulation were considered first, but after investigating them it was concluded that they were impractical for high-fidelity television transmission. For instance, plate modulation, used so universally for sound broadcasting, requires high peak-to-peak modulating voltages. When the modulating signal wave changes with almost infinite abruptness, due to the process of scanning, the effective capacitances between the radio-frequency plate circuit and ground become practically short circuits. Coupling resistors of low value are necessary to maintain a satisfactory frequency response.[1] As a result video-frequency modulator tubes operate at low plate efficiencies and the power ratings of the tubes required appear to be unreasonably large when the antenna power is considered.

Turning to grid modulation, we find that it has the advantage of requiring lower modulating voltages, but the plate efficiency of a grid-modulated radio-frequency amplifier is inherently low.

A third method is absorption modulation. This little-used scheme is characterized by low efficiency and low percentage modulation. When vacuum tubes are employed as absorbers they usually extract energy from a tank circuit coupled to the transmitter load, so that they operate upon the principle: the power left for the load is that which the modulators do not absorb.

Now these conventional modulation methods all have the fundamental defect, when used for television, that their operation depends upon changing the radio-frequency voltage amplitude across a tank circuit. The inherent flywheel action in such a circuit effectively prevents rapid variations in the voltage envelope.

In order to avoid serious side-band clipping, it is customary to damp the circuit by overcoupling. The result is low plate efficiency. For instance, it is reported that one grid-modulated television transmitter operating at 50 megacycles has an input to the modulated amplifier of 60 kilowatts when the antenna power[2] is only 7.5 kilowatts.

Taking into consideration tank-circuit decrement, let us see by the aid of Fig. 1 how limited we are in peak power output. Plotted against modulation frequency in megacycles are shown calculated values of peak power output for several ultra-high-frequency power tubes. The sloping portion of the curves was calculated from the approximate ex-

[1] R. D. Kell, A. V. Bedford, and M. A. Trainer, "An experimental television system—the transmitter," Proc. I.R.E., vol. 22, pp. 1246–1265; November, (1934).

[2] J. W. Conklin and H. E. Gehring, "Television transmitters operating at high power and ultra-high frequency," *RCA Rev.*, vol. 2, pp. 34–37; July, (1937).

pression (see Appendix I)

$$P = \frac{96.2 I_B^2}{f_m C} \text{ (kilowatts per tube)}$$

where,

I_B = maximum direct plate current in amperes
f_m = maximum modulating frequency in megacycles
C = total effective plate-to-ground capacitance (including neutralizing condensers) in micromicrofarads.

The horizontal portion[3,4] of the curves indicate the output to be expected at 50 megacycles.

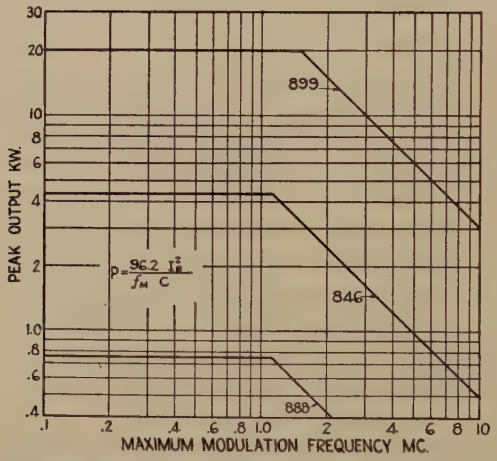


Fig. 1—Theoretical curves of peak output against modulation frequency for various tubes.

Let us take as an example the type 899 tube rated at 30 kilowatts plate dissipation. At 4 megacycles top modulating frequency, the theoretical peak power would be 7.6 kilowatts per tube, corresponding to a nominal carrier of only 3.8 kilowatts for a pair of these tubes operated as modulated amplifiers.

The conclusion reached from considering conventional methods of modulation was that something new and different was needed if the requirements for television transmission were to be met. The first step was the query: If the decrement of the tank circuit causes so much trouble, why not modulate between the tank and the antenna? The problem then was to find a suitable arrangement for doing this without

[3] W. C. White, *Gen. Elec. Rev.*, vol. 36, pp. 394–397; September, (1933).
[4] John Evans, "Ultra-high-frequency high-power transmitter using short transmission lines," presented before Eleventh Annual Convention, Cleveland, Ohio, May 11, 1936.

the use of even more tank circuits with their attendant side-band clipping. The solution was reached in what has now become known as "transmission-line modulation."

Transmission-Line Modulation

In an early test a pair of wires was connected to the 2-wire transmission line joining the radio-frequency oscillator and the antenna at the point J shown in Fig. 2. The length of these wires corresponded to a quarter of a wavelength. The generator output to the load fell practically to zero when the ends of these wires were open. This showed that the quarter-wave line was behaving according to theory, because the open-end condition reflected back as a short circuit at point J. The next step was to choose point J one quarter of a wavelength from the generator, so that the effect of short-circuiting the transmission line at

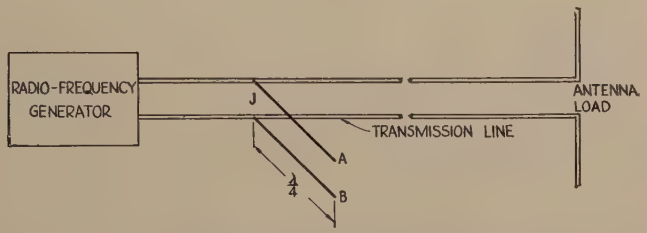


Fig. 2—The transmission-line modulation circuit.

J appeared as an open circuit to the generator. These quarter-wave sections can well be called "impedance inverters" since this is what they actually do in a transmission-line modulation system.

Now when a shorting bar was placed across AB, or the tuning of this section varied, the output promptly went up. Connecting various values of carbon resistors across the open ends AB resulted in various outputs corresponding to the resistance values. To enable modulation by a television signal, vacuum-tube modulators replaced the resistors. These were arranged as shown in Fig. 3. Here the schematic circuit for the radio-frequency generator is also shown. In fact this is a diagram of a typical transmission-line modulated transmitter, except for the power-supply and video-frequency amplifiers.

There is no direct-current plate supply to the modulator tubes, although the plates have a direct-current return path to the filaments through the transmission lines. The radio-frequency generator employs the usual push-pull oscillator circuit adjusted for optimum output, with the modulator line L_m disconnected from the feeders. The physical

length of L_m will be shorter than a quarter wavelength, due to the plate capacitance of the modulator tubes.

The operation of the transmission-line modulator can be understood in connection with Fig. 3. Normally the radio-frequency voltage impressed on the modulator plates will cause a small amount of rectified plate current to flow to the filaments, thus placing a finite resistance across the end of line L_m. If the modulator grids are made sufficiently negative no plate current will flow. This corresponds to zero conduct-

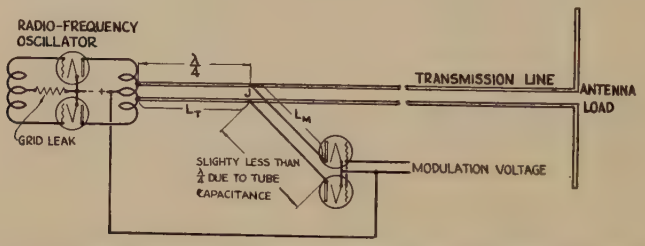


Fig. 3—Schematic diagram of a transmission-line modulated transmitter.

ance, or an open circuit at the end of line L_m, resulting in zero output to the transmission line. As the grids are made less negative, the output will rise, until finally peak generator output will be obtained.

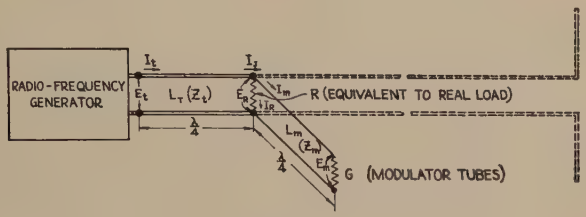
Design Considerations

In order to design a transmission-line modulation system it is necessary to know the various currents and voltages involved. As a matter of convenience the various current and voltage relations can be found in terms of m, which denotes that ratio between actual load current and the load current with the line L_m disconnected from the antenna feeders.

Let us refer to Fig. 4, where the essential elements of a transmission-line modulation system are shown. A radio-frequency generator of substantially constant voltage E_t is connected to load R through a quarter-wave impedance inverter L_t, which has a surge impedance Z_t. Across this load is also connected a second quarter-wave impedance inverter L_m, having a surge impedance Z_m. The far end of L_m is terminated in a conductance G, which represents the effective conductance of the plate-filament circuits of the modulator tubes.

The working formulas shown in Fig. 4 give the voltage, current, and power relations at the generator output, at the load R, and at the modulator, respectively. They were derived from the fundamental principle that the ratio of sending-end voltage to receiving-end current is equal to the surge impedance for a low-loss quarter-wave line. For constant E_t and Z_t it follows that at the junction of the two quarter-

wave inverters and the load, the generator acts as a constant-current source.



$$I_i = \frac{E_t}{Z_t}$$

$$I_t = \frac{E_t R}{Z_t^2} m \qquad I_R = \frac{E_t}{Z_t} m \qquad I_m = \frac{E_t}{Z_t}(1 - m)$$

$$E_t = \text{(assume constant)} \qquad E_R = \frac{E_t R}{Z_t} m \qquad E_m = \frac{E_t Z_m}{Z_t}(1 - m)$$

$$P_t = \frac{E_t^2 R}{Z_t^2} m \qquad P_R = \frac{E_t^2 R}{Z_t^2} m^2 \qquad P_m = \frac{E_t^2 R}{Z_t^2} m(1 - m)$$

$$\mathcal{E}_m^{FF} = \frac{P_R}{P_t} = m \qquad G = \frac{Rm}{Z_m^2(1 - m)}.$$

Fig. 4—Design formulas for idealized transmission-line modulation system.

Some of the above relations are shown graphically in Fig. 5. For convenience, power values are given in per cent of maximum output ($m = 1$).

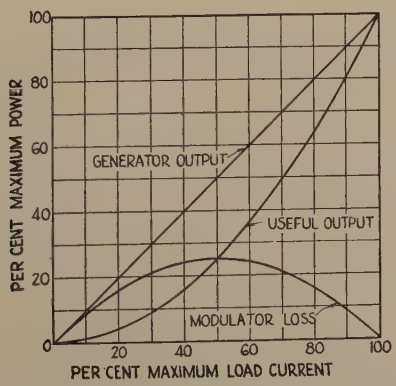


Fig. 5—Curves showing power relations in a transmission-line modulated transmitter.

The nominal carrier power is, of course, one quarter the peak output at $m = 0.5$, and it is interesting to note that the modulator dissipation is a maximum at this point and equal to the carrier. For any appreciable modulation, then, the modulator loss averaged over a

modulation cycle will be less than one quarter the peak transmitter output. This decrease of modulator loss under modulation is quite apparent in practice.

So far the design considerations have been based on ideal assumptions of loss-free quarter-wave inverters and modulator tubes capable of an effective plate resistance of zero. In addition, if the load current is to be linear with respect to the voltage on the modulator grids, the modulator conductance G must also bear a certain relation to this voltage as indicated by the nonlinear factor $m/(1-m)$ in the formula for G (Fig. 4).

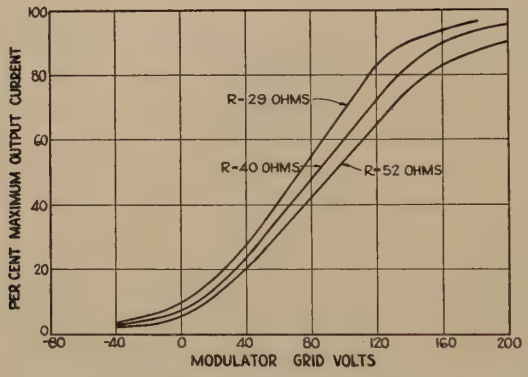


Fig. 6—Typical modulation characteristic curves for loads of various resistances.

Optimum results for a given modulator tube depend upon the proper choice of load resistance R and the surge impedances Z_t and Z_m. If R is too low, the ohmic, radiation, and dielectric losses in L_m and its associated tube-plate leads will prevent it from presenting a good enough short circuit to stop current from flowing into R. On the other hand, if R is chosen too high, the useful peak output will be limited because the modulator tube conductance cannot be sufficiently increased. Typical experimental modulation characteristics illustrating the effect of the choice of R are shown in Fig. 6. The deep modulation capabilities are obvious and it is also interesting to note that the output current is quite linear with respect to modulator grid voltage, showing that the proper conductance—grid-voltage relation fortunately exists in standard triodes. The coupling to the oscillator is ordinarily increased until the upper bend of the characteristic is due chiefly to oscillator overload so that the practical peak oscillator output is closely approached. This effect is clearly shown by the upper curve.

It is interesting to note that about 150 volts, peak to peak, of video

frequency are required for modulation, and that the input capacitance of the modulator grids is only 32 micromicrofarads. A typical 1-kilo-

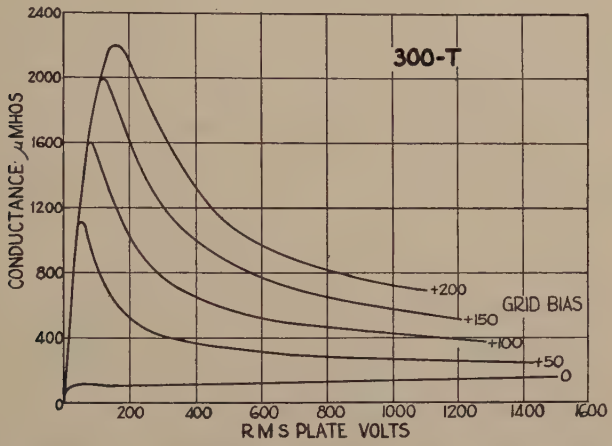


Fig. 7—Family of effective-conductance curves for the Eimac 300T tube used as modulator.

watt grid-modulated radio-frequency amplifier would require a video-frequency swing of the order of 1000 volts into a capacitive load of at least 100 micromicrofarads.

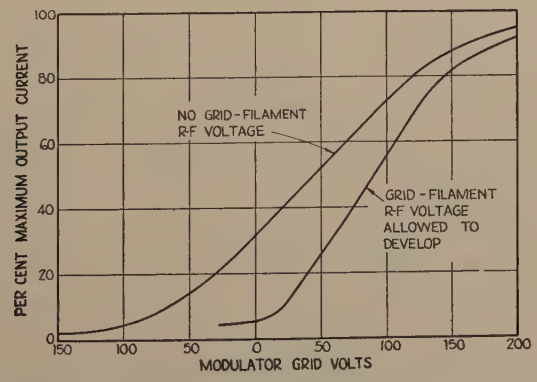


Fig. 8—Modulator curves showing the effect of radio-frequency voltage applied between grids and cathodes of the modulator tubes.

Modulation characteristics for a transmission-line modulation system can be calculated with fair accuracy in the maximum output region by the use of modulator-conductance curves. Fig. 7 shows a family of such curves for the Eimac 300T tube, which were calculated by as-

suming a given sinusoidal plate voltage and direct-current grid bias, and then determining the fundamental component of plate current.

Calculated modulation characteristics usually indicate a higher negative grid voltage than is really needed to secure minimum output. This is partly due to the fact that in this region the modulator radio-frequency circuit losses become relatively important. Another factor which can be made to reduce materially the modulating voltage required is the *radio*-frequency voltage allowed to develop between modulator grid and filament. An example of this is shown in Fig. 8.

Advantages of New Method

With such a new method of modulation available it is well to examine it from a different angle. In the first place, transmission-line modulation compares very favorably in over-all plate efficiency with typical audio-frequency modulation systems as shown in Table I. Only high-level class B plate modulation gives a higher theoretical efficiency. The extreme simplicity of equipment necessary for transmission-line modulation may be advantageous in a number of ultra-high frequency sound-transmitter applications.

TABLE I

Type of Transmitter	Plate Modulation		Linear Radio-Frequency Amplifier	Grid Modulation	Transmission-Line Modulation
	Class A	Class B			
Antenna Carrier (kw)	1	1	1	1	1
Radio-frequency generator, unmodulated output (kw)	1	1	1	1	2
Radio-frequency generator, peak output (kw)	4	4	4	4	4
Radio-frequency generator, plate efficiency (%)	66	66	33	20	66
Radio-frequency generator, plate input (kw)	1.5	1.5	3.0	5.0	3.0
Modulator output (100% mod.) (kw)	0.75	0.75	—	low	—
Modulator plate efficiency (%)	30	60	—	—	50
Modulator plate input (kw)	2.5	1.25	—	—	—
Total plate input (kw)	4.0	2.75	3.0	5.0	3.0
Over-all plate efficiency (%)	25	36.5	33	20	33

Second, transmission-line modulation is inherently a wide-band system, because the stored energy undergoes a relatively small change over the modulation cycle. For a suddenly applied load the increased power demanded will be instantly supplied by the tank circuit itself, and the radio-frequency tank voltage will gradually drop a few per cent until the tube output becomes adjusted to the new load. This effect should correspond to a radio-frequency source of low impedance for rapid load changes, and the generator impedance should be ap-

preciable only for modulation frequencies below a certain value, dependent upon the tank decrement. This was verified experimentally, by first measuring the depth of modulation of the radio-frequency voltage across the generator tank and then that delivered to the load in a typical transmission-line-modulated transmitter. The ratio of the two for various modulation frequencies is plotted in Fig. 9. The obvious falling off at about 1 megacycle checked well with the estimated Q of the radio-frequency generator tank circuit. The assumption of constant E_t at the generator is therefore justified at the higher modulation frequencies.

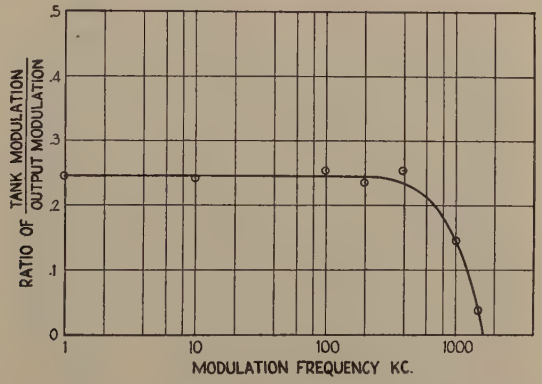


Fig. 9—Curve of ratio of tank modulation to output modulation plotted against frequency.

With this new system the limitations of load and stray capacitance are removed and the radio-frequency generator problem is greatly simplified. The generator can now be adjusted for the greatest peak power under optimum conditions, and the plate voltage raised until dissipation or excess radio-frequency currents become the limit. The 899 tubes are now capable of a peak power of 20 kilowatts per tube at 50 megacycles, as indicated in Fig. 1. This is an increase of 2.5 times the power in the previous example. High power necessary for first-class television coverage is now possible by ordinary parallel operation of existing tubes, or by the use of other methods to overcome the "size-wavelength" limitation of transmitting tubes.

A Typical Installation

The Philco high-fidelity television transmitter W3XE successfully uses transmission-line modulation. The peak carrier power is 4 kilowatts, giving a nominal carrier of 1 kilowatt. Linear modulation up to 80 per cent is obtained.

The radio-frequency generator consists simply of two type-846 water-cooled oscillators mounted in the upper ends of large brass tubes forming the "organ-pipe" plate tank circuit shown in Fig. 10. The grid tank is somewhat smaller and extends horizontally out from the grid terminals. The oscillator frequency is determined by the positions of both grid and plate shorting straps while the excitation can be adjusted by changing the ratio of the tank lengths. The oscillator is operated conservatively with an average plate input of 4.3 kilowatts from a 5-kilovolt rectifier. For an average carrier power of 1 kilowatt the

Fig. 10—The power oscillator W3XE. The tubes are placed at the top of the "organ-pipe" plate circuit. The grid circuit extends horizontally.

high-level plate efficiency is thus 23 per cent, which is quite reasonable, considering that a self-excited oscillator is used.

The frequency stability of W3XE has been quite good considering that no special high-Q circuits have been employed. The day-to-day carrier frequency does not shift more than a few kilocycles as checked on a heterodyne frequency meter. When modulating rather deeply with a low-frequency square wave the carrier "wobbulation" is conveniently checked by observing on the frequency monitor the extra beats on either side of the carrier. These ordinarily are within audible range, indicating negligible frequency variation as compared with the channel width of 6 megacycles.

In the usual transmitter, one cause of frequency "wobbling" under modulation is the change in radio-frequency grid and plate voltages

because the effective tube input and output capacitances depend upon the voltage ratio. With transmission-line modulation these voltages remain essentially constant so that frequency "wobbling" is minimized. Although the load on the oscillator tank varies it is always resistive. At minimum output L_t reflects back a resistive open circuit to the oscillator. At maximum output the load is connected directly to the tank and is usually of high power factor. The elimination of the radio-frequency exciter and neutralized power amplifier further simplifies the ultra-high-frequency transmitter problem.

The load is connected directly across a few inches of the plate tank and isolated with mica blocking condensers, as shown in Fig. 11. The

Fig. 11—Lower end of oscillator plate tank circuit, showing the blocking condensers feeding the load.

line to the junction is a quarter wave long and of rather low surge impedance. A load-matching line transformer, also a quarter wave long, extends vertically from the modulation junction out the top of the transmitter. After passing through a network which combines the output of a separate 200-watt sound transmitter, the signals are fed into a balanced concentric line leading to a wide-band antenna system supported 210 feet above the street, as shown in the photograph of Fig. 12.

The modulation system shown in Fig. 13 consists of a relatively high-impedance line between the junction and the modulator tubes whose plates are arranged in parallel push-pull. The four tubes in the photograph are Eimac 450TL tubes adjusted so as to dissipate about 250 watts per tube.

The modulator grids are directly coupled to a 2-stage amplifier whose response extends from zero to 5 megacycles. An additional direct-coupled amplifier stage (which can be switched in while operating) provides negative polarity of modulation when desired. The power to

the video-frequency amplifier system is supplied from the same 5-kilovolt rectifier as the oscillator through a special voltage-divider network. The entire transmitter (except for power supply) is enclosed in a galvanized iron box having separate compartments for the oscillator, modulator, and controls.

After several years of experience with transmission-line modulation in numerous arrangements, it can be said that results have been very satisfactory. At no time could a suppression of side bands be attributed to the system of modulation. Actual measurement in the field, on

Fig. 12—Tower and antenna arrays for W3XE, 210 feet above the street.

W3XE, shows good side-band response up to a 5-megacycle modulation frequency, thus permitting the radio transmission of high-definition 441-line television pictures.

Conclusions

It can be concluded that transmission-line modulation is just as efficient as conventional methods for audio-frequency modulation, and is definitely superior where video modulation frequencies are involved. By removing the tank-decrement limitation, greater peak power and plate efficiencies are possible for a given tube. This fact, combined with

the possibility of parallel operation, means that transmission-line modulation will provide a greater television-transmitter power than any other known system. In addition, the transmitter can be made simpler and consequently less expensive. It is hoped that these advantages will bring us one step closer to high-fidelity television pictures in our homes.

Appendix I

The peak power output to be expected from a given tube operating into a highly damped tank circuit will now be derived.

The output is equal to I^2r watts, where I represents the fundamental component of plate current and r the effective load resistance across

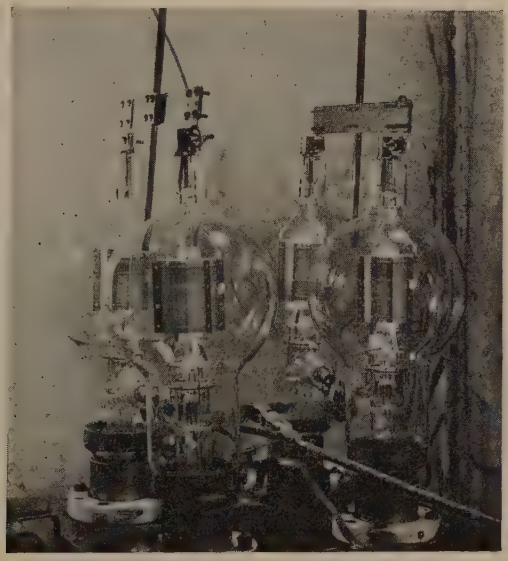


Fig. 13—Modulator tubes. Eimac type 450TL.

the plate tank circuit. The root-mean-square plate current for half sine waves is a function of peak instantaneous plate current, which, for the larger power triodes, is limited by total emission. The maximum rated I of a particular tube will be taken as 0.707/0.636 or 1.1 times the maximum average direct plate current recommended for class C operation.

The load resistance r is limited in turn by the circuit capacitances and band width. If a radio-frequency amplifier were connected to a load resistor r, very poor operation would be secured due to the relatively high displacement currents in the tube capacitances. Connecting an inductive tank reactance X across r of the proper value would correct this low-power-factor difficulty and at resonant frequency the load

would consist of r alone, neglecting circuit losses. Off resonance, the correction would not be complete so that lowered output would again result. The lower the ratio r/X, the further off resonance would occur a given limit of poor operation. As a matter of convenience, the frequency-deviation limit will be taken where the resistive and reactive components of the plate current are equal.

For a resistance, inductance, and capacitance in parallel, the combined susceptance is

$$b = \frac{1}{X_c} - \frac{1}{X_1} . \tag{1}$$

If $X_1 = X_c = X$ at frequency f, at some other frequency such as $f \pm f_m$, the susceptance becomes

$$b = \frac{1}{X}\left(\frac{f \pm f_m}{f} - \frac{f}{f \pm f_m}\right) = \pm \frac{f_m(2f \pm f_m)}{fX(f \pm f_m)} . \tag{2}$$

The resistive and reactive components of the combined admittance become equal when $r = 1/b$ or

$$r = \pm \frac{f(f \pm f_m)}{f_m(2f \pm f_m)} X. \tag{3}$$

Expression (3) applies if f is taken as the carrier frequency and f_m is the highest modulation frequency. When f /f is relatively low, (3) simplifies to

$$r = \frac{f}{2f_m} X. \tag{4}$$

This expression is closely related to the usual formula involving the Q of a circuit and band width.

At ultra-high frequencies the tube and neutralizing capacitances are usually sufficient for the tank circuit without additional capacitance. The reactance X then becomes the reciprocal of $2\pi f$ times the effective output capacitance C of the amplifier to neutral. Substituting in (4),

$$r = \frac{1}{4\pi f_m C} . \tag{5}$$

The output power in kilowatts now becomes

$$P = \frac{I^2 r}{1000} = \frac{(1.1 I_b)^2 \times 1000}{4\pi f_m C} = \frac{96.2 I_b^2}{f_m C}, \tag{6}$$

for I_b in amperes, f_m in megacycles, and C in micromicrofarads.

Appendix II

The equipment used to demonstrate transmission-line modulation consists of four type-834 tubes connected as shown in Fig. 3, except

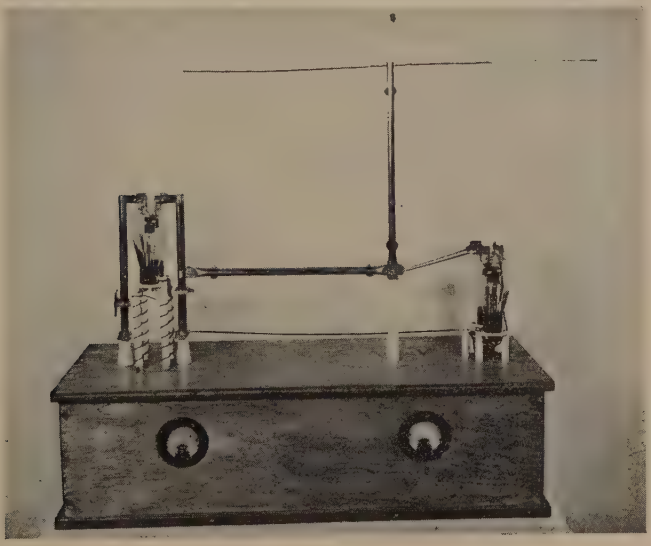


Fig. 14—Working model used to demonstrate transmission-line modulation. The carrier frequency is 200 megacycles.

that radio-frequency chokes are used in the oscillator-filament leads to facilitate operation at 200 megacycles. A convenient means of indicating output consists of a small lamp connected across the input terminals of the half-wave antenna used as the load. The transmission line from the antenna to the modulator junction is also a quarter-wave long for compactness.

The modulator tubes are on the right in the photograph of Fig. 14 and the modulator line is only about six inches long to the plate terminals. The compartment under the transmitter contains a 500-volt plate supply for the oscillator, and a source of modulation voltage which supplies direct current, 1000 cycles, 1.5, 5, 10, and 20 mega-

cycles to the modulator grids. A self-contained tube voltmeter enables constant output to be maintained. All frequencies above 1000 cycles are also modulated by the 1000-cycle tone for checking frequencies with a standard all-wave receiver.

With the modulator system made ineffective by shorting the plates, the transmitter is adjusted for optimum coupling, which in this case is maximum output into the antenna.

With the modulator short circuit removed, but with the tubes still unlighted, the transmitter frequency is next adjusted to the resonant frequency of the modulator line, as indicated by practically no antenna output. Detuning the modulator line with hand capacitance brings the output up again.

The modulator tubes are next lighted and the direct-current grid bias varied up to $+22\frac{1}{2}$ volts with corresponding increases in output as indicated by the lamp. With the bias set at the center of the modulation characteristic various frequencies are impressed. As indicated by an increase in output the modulation is upward for frequencies up to and including 20 megacycles.

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HIGH-EFFICIENCY MODULATION SYSTEM*

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Summary—*A descriptive and mathematical treatment is given of a means for obtaining modulated radio-frequency power with good conversion efficiency from the usual direct-current source. The method employed is to modify the load line on a saturated radio-frequency amplifier to take care of positive peaks and to grid modulate the amplifier for negative peaks. Load modification is accomplished by absorption and in this system the absorbed power is not dissipated in heat but is returned to the direct-current source for the power-amplifier tube and thereby reduces the drain from the supply. This results in direct-current-to-carrier conversion efficiencies in the order of 50 to 60 per cent when average tubes are used.*

THERE have appeared in the past three high-efficiency modulation systems of particular merit. One is the "outphasing" system of Chireix.[1] The second is the high-level modulation system employing class B audio-frequency modulators as described by Chambers, Jones, Fyler, Williamson, Leach, and Hutcheson.[2] The third is the ingenious quarter-wavelength section coupled amplifier described by Doherty.[3]

These systems have been developed to reduce the power consumption or the tube complement of broadcast transmitters. The favored method of providing large modulated outputs in the past has been that of employing linear radio-frequency power amplifiers to amplify the modulated output of some lower-powered unit. This system yields direct-current-to-carrier conversion efficiencies in the order of from 25 to 35 per cent.

The other systems referred to yield direct-current-to-carrier efficiencies in the order of 50 to 70 per cent which means that their use results in power savings of about one half. The system to be described in this paper will yield efficiencies in the order of from 50 to 60 per cent for most tubes.

* Decimal classification: R148×R355.8. Original manuscript received by the Institute, January 12, 1938. Presented before New York Meeting, January 5, 1938.

[1] Chireix, "High power outphasing modulation," Proc. I.R.E., vol. 23, pp. 1370–1392; November, (1935).

[2] Chambers, Jones, Fyler, Williamson, Leach, and Hutcheson, "The WLW 500-kilowatt broadcast transmitter," Proc. I.R.E., vol. 22, pp. 1151–1180; October, (1934).

[3] Doherty, "A new high-efficiency power amplifier for modulated waves," *Bell Sys. Tech. Jour.*, vol. 15, pp. 469; July, (1936); Proc. I.R.E., vol. 24, pp. 1163–1182; September, (1936).

The method employed is that of modifying the load line on a saturated radio-frequency amplifier to take care of positive peaks of modulation and to grid modulate the amplifier to take care of negative peaks of modulation. Load modification is accomplished by a modulation-controlled absorber and in this system the absorbed power is not dissipated in heat but is returned to the direct-current source of supply for the power amplifier. This reduces the net power drain from the supply source itself and hence results in the efficiencies mentioned. In order to present the theory more clearly, consideration will be given to actual schematic diagrams.

The Basic Circuit

The basic circuit is shown in Fig. 1. The power amplifier tube 1, which may be neutralized if necessary, has its anode connected to the

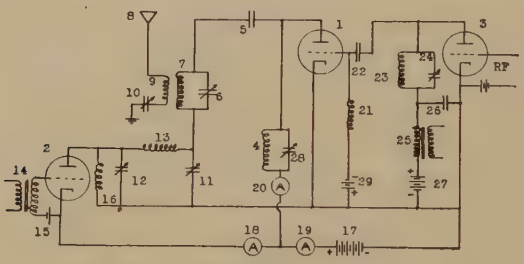

Fig. 1

direct-current supply through the impedance formed by inductor 4 in shunt with capacitor 28. The anode is also connected to blocking capacitor 5, thence through the parallel tuned circuit consisting of inductor 7 and capacitor 6, and thence through a quarter-wavelength section consisting of capacitors 11 and 12 and inductor 13 to cathode. The load circuit shown consists of the antenna 8 with its tuning elements, capacitor 10 and inductor 9. Inductor 9 also serves as a coupling means between the load circuit and the anode inductor 7. Choke 16 serves to carry the direct-current component of the modulator-rectifier tube 2. Instead of calling this tube a modulator-rectifier we may abbreviate it by dropping the last six letters of modulator and the first four letters of rectifier leaving the word modifier.

The anode of the modifier is connected to the high end of capacitor 12, and the cathode of the modifier is connected to the positive terminal of the plate supply through ammeter 18 which indicates the plate current being furnished by the modifier and ammeter 19 which indicates the plate current being furnished by the direct-current plate-supply

source 17. Ammeter 20 indicates the direct-current component of the plate current to tube 1. The grid of the modifier is connected through the secondary of audio-frequency transformer 14 to the bias supply 15 and thence to the cathode of the modifier. The audio frequencies with which the carrier is to be modulated are supplied to the primary of transformer 14.

The grid of tube 1 is connected through radio-frequency choke 21 and bias supply 29 to the cathode of tube 1. Also connected to the grid is coupling capacitor 22 which in turn is connected to the output tank circuit of exciter tube 3 consisting of inductor 23 in shunt with capacitor 24. The lower end of this tank is by-passed to the ground by capacitor 26, and this exciter is plate modulated through audio-frequency transformer 25, the lower end of which is connected to the positive terminal of plate supply 27 for exciter tube 3. The grid of exciter tube 3 is excited from a suitable radio-frequency means such as an oscillator or radio-frequency amplifier of adequate size to saturate the grid of tube 3 so that plate modulation may be effected.

The circuit is adjusted as follows. Capacitor 5 is disconnected at the point where it connects to capacitor 6 and capacitor 28 is adjusted to yield minimum direct plate current for tube 1. In this way the tube capacitance is tuned out. Choke 4 is a large choke yet small enough so that it may be tuned by a small trimmer condenser 28 together with the anode-to-ground and stray capacitance in the anode wiring. The next step is to reconnect condenser 5 to capacitor 6 and to short-circuit capacitor 11 and tune capacitor 6 to give a minimum plate current in tube 1. The load circuit is next tuned to resonance by capacitor 10 and the coupling between inductors 9 and 7 adjusted until the antenna current is double that required under carrier conditions. While the above adjustments are being made it is essential that the grid of tube 1 be fairly well saturated.

The next step is to adjust the quarter-wavelength section. This is accomplished by removing the short on capacitor 11 and placing a short circuit across capacitor 12 and then tuning capacitor 11 for *minimum* plate current in tube 1. This insures that the reactances of inductor 13 and capacitor 11 are equal but opposite in sign, which is one of the necessary relationships for the quarter-wavelength section. The next step is to remove the short circuit across capacitor 12 and to bias tube 2 beyond cutoff, and then adjust capacitor 12 until the plate current of tube 1 is a *maximum*. This insures that the reactances of inductor 13 and capacitor 12 are equal but opposite in sign, which is the second necessary relationship for the quarter-wavelength section. The final step is to allow tube 2 to rectify by dropping bias 15 to

such a value that the antenna current assumes carrier value. The excitation on tube 1 should now be adjusted for carrier conditions which is not a saturated grid but one which just gives full excursion to plate-voltage swing on tube 1.

The transmitter is now ready for operation. For positive peaks of modulation the grid of tube 2 should be driven negative to cutoff and the plate of tube 3 should be driven positive by the audio frequencies. For negative peaks the plate of tube 3 is driven below the direct supply voltage 27 in the usual way and tube 1 performs as a class B radio-frequency amplifier. It is thus seen that both positive and negative modulation is cared for.

Mathematics of the Basic Circuit

Let R represent the effective load across the tank consisting of inductor 7 and capacitor 6. Since full plate swing is in effect for both the carrier condition and positive peaks of modulation, the only way to reduce the swing across R to one half for carrier conditions is to insert a resistance of R ohms in series with the tank. If we refer to that end of the quarter-wavelength section nearest the tank as the near-end of the section, then under carrier conditions the near-end impedance must be R ohms. The far-end impedance will then be

$$R_{12} = \frac{Z_0^2}{R} \tag{1}$$

where Z_0 is the surge impedance of the quarter-wavelength section and R_{12} is the effective alternating-current resistance presented across capacitor 12 by the modifier and its direct-current load circuit.

The direct-current power being furnished by the modifier is equal to the carrier power under carrier conditions. Since the modifier direct-current output voltage is E_b, where E_b is the voltage of the plate supply 17, the direct current being delivered by the modifier must then be

$$I_{18} = \frac{W_c}{E_b} \tag{2}$$

where W_c is the radio-frequency carrier power.

It is well known that in such peak rectifiers the equivalent alternating-current resistance presented to the alternating-current circuit is approximately half the direct-current effective resistance, and since this is the far-end resistance, this becomes

$$R_{ac} = \frac{R_{dc}}{2} = \frac{E_b}{2I_{18}} = \frac{E_b^2}{2W_c} = R_{12}. \tag{3}$$

A more exact relationship between R_{ac} and R_{dc} is given in Appendix IV. Equation (3) may be substituted in (1), and Z_0 solved for, whence

$$Z_0^2 = RR_{12} = \frac{RE_b^2}{2W_c}$$

and

$$Z_0 = E_b\sqrt{\frac{R}{2W_c}}. \tag{4}$$

Now the surge impedance is given by

$$Z_0 = \sqrt{\frac{L_{13}}{C_{11}}}. \tag{5}$$

But since $\omega L_{13} = 1/\omega C_{11}$, (5) becomes by substitution

$$Z_0 = \omega L_{13}. \tag{6}$$

Substituting (6) in (4) there results

$$\omega L_{13} = E_b\sqrt{\frac{R}{2W_c}} \tag{7}$$

or

$$L_{13} = \frac{E_b}{\omega}\sqrt{\frac{R}{2W_c}}. \tag{8}$$

This permits the inductor 13 and the capacitors 11 and 12 to be calculated in advance of their incorporation in the circuit.

The maximum voltage to which L_{13} will be subjected may be approximated by assuming that the peak plate swing of tube 1 is equal to kE_b, where k usually lies between 0.85 and 0.95. The *maximum* plate current is then

$$I_{max} = \frac{kE_b}{R}. \tag{9}$$

Assuming the current to be half sine waves, the root-mean-square value of the fundamental will be

$$I_{rms} = \frac{I_{max}}{2}. \tag{10}$$

Substituting (9) in (10),

$$I_{rms} = \frac{kE_b}{2R}. \tag{11}$$

This current flows through L_{13} on positive peaks of modulation, hence the voltage across L_{13} is given by

$$E_{13} = I_{\text{rms}}\omega L_{13} = \frac{kE_b^2}{2R}\sqrt{\frac{R}{2W_c}}. \tag{12}$$

But the carrier-power ouput is

$$W_c = \frac{k^2E_b^2}{16R}. \tag{13}$$

Substituting this in (12) for W_c,

$$E_{13} = \sqrt{2}\,E_b \quad \text{r-m-s volts} \tag{14}$$

or

$$E_{13} = 2E_b \qquad \text{peak volts.} \tag{15}$$

The same voltage will exist across capacitor 12. This information permits the proper design of these units.

The choke 16 must also stand this voltage and the actual capacitance of capacitor 12 must be adjusted so that capacitor 12 in shunt with choke 16 presents a net *negative* reactance equal to that of (7). It is also possible to make L_{13} adjustable if desired either by an adjustable iron core, by a variometer scheme, or by the use of an adjustable capacitor in shunt with L_{13}.

The Q of L_{13} should be high so that not too much power is dissipated in this coil. If under positive peaks of modulation we do not wish this loss to exceed 2 per cent of the total power, then its resistance should be not greater than 2 per cent of R. If this decimal resistance be called dR, then the Q required of L_{13} is

$$Q_{13} = \frac{\omega L_{13}}{dR} = \frac{E_b}{dR}\sqrt{\frac{R}{2W_c}}. \tag{16}$$

Substituting (13) in (16),

$$Q_{13} = \frac{2\sqrt{2}}{dk}. \tag{17}$$

Assuming $k=1$ and $d=0.02$, then Q must be

$$Q_{13} = \frac{2\sqrt{2}}{0.02} = 140. \tag{18}$$

This is not an unreasonable value to obtain in practice.

Conversion Efficiency

The conversion efficiency of this system of modulation may be calculated as follows. The carrier output is given by (13) and the *tube* input is given by

$$W_{in} = \frac{E_b I_{\max}}{2\pi} . \tag{19}$$

Hence the *tube* efficiency is given by twice (13) divided by (19), or

$$\text{tube eff.} = \frac{2W_c}{W_{in}} = \frac{2k^2E_b{}^2 2\pi}{16RE_bI_{\max}} . \tag{20}$$

But by substituting the $RI_{\max}$ of (9),

$$\text{tube eff.} = \frac{k^2\pi E_b}{4kE_b} = \frac{k\pi}{4} . \tag{21}$$

The anode loss is given by (19) minus (13) which is

$$W_L = W_{in}\left(1 - \frac{k\pi}{4}\right). \tag{22}$$

It is thus seen that where $k=1$ the *tube* efficiency becomes $\pi/4$ or 0.785 or 78.5 per cent and the anode loss $1-\pi/4$ or 21.5 per cent of the tube input. This means that a tube operated under this system can deliver a carrier power of one half the class C rating of the tube without in any way exceeding tube ratings.

Of the total plate input, not all comes from the plate supply, for the modifier furnishes W_c watts of this power. Therefore the conversion efficiency as far as the direct-current plate supply is concerned is

$$\textit{power} \text{ eff.} = \frac{W_c}{W_{in} - W_c} . \tag{23}$$

Substituting (13) for W_c and (19) for W_{in}

$$\textit{power} \text{ eff.} = \frac{\dfrac{k^2E_b{}^2}{16R}}{\dfrac{E_bI_{\max}}{2\pi} - \dfrac{k^2E_b{}^2}{16R}} . \tag{24}$$

Substituting (9) in (24) for $I_{\max}$,

$$\textit{power} \text{ eff.} = \frac{\dfrac{k^2E_b{}^2}{16R}}{\dfrac{kE_b{}^2}{2\pi R} - \dfrac{k^2E_b{}^2}{16R}} = \frac{1}{\dfrac{8}{k\pi} - 1} . \tag{25}$$

Thus the *power* or *conversion* efficiency of this system with ideal tubes is (25) with $k=1$, or

$$\text{ideal } \textit{power} \text{ eff.} = \frac{1}{\frac{8}{\pi} - 1} = 64.6\%. \tag{26}$$

Fig. 2 shows the carrier-conversion efficiency as a function of k, the percentage of the plate-supply voltage that the peak plate voltage

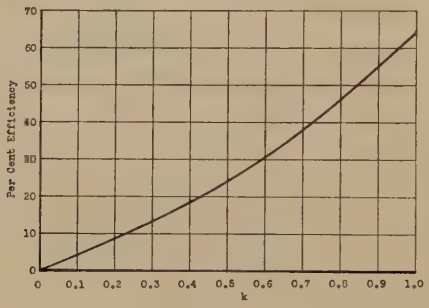

Fig. 2

swing is. This factor may be readily determined from tube curves. Some typical efficiencies which may be expected from a few tubes in commercial use are shown in Table I.

TABLE I

Tube Type	Carrier Power	Tube Input	Tube Eff. %	Anode Loss	E_b	Conversion Efficiency %
862	50,000	139,500	71.7	39,500	18,000	56.2
858	10,000	27,200	73.5	7,200	18,000	58.0
863	5,000	13,400	74.7	3,400	14,000	60.1
1652	2,500	6,820	73.3	1,820	7,500	58.0
846	1,250	3,560	70.2	1,060	7,500	54.0
851	750	2,080	72.1	580	2,500	56.2
806	200	545	73.3	145	3,000	58.0
805	100	273	73.3	73	1,500	58.0
203A	50	138	72.5	38	1,250	56.5
808	50	135	74.0	35	1,500	58.5
807	12.5	38.6	64.8	13.6	400	48.0

It is seen that practically all of the important tubes lie in the region between 55 and 60 per cent efficiency with a mean at 57.5 per cent indicating that $k=0.925$ on the average.

Modifier Considerations

The modifier rating required for a given power amplifier may be calculated. First, there is the question of peak current through the modifier. In order to solve for this it is necessary to solve for the net

maximum voltage applied between cathode and anode. This is the voltage difference between the peak voltage across the far end of the quarter-wavelength section and the direct voltage of the plate supply. In Appendix I it is shown that the ratio of modifier plate resistance r_p to the equivalent direct-current load resistance R_{dc} is

$$\frac{r_p}{R_{dc}} = \frac{\cot \beta - \left(\frac{\pi}{2} - \beta\right)}{\pi} \tag{27}$$

where,

$$\beta = \sin^{-1} \frac{E_b}{E_p}$$

where,

E_p = peak alternating voltage across far end of quarter-wavelength section.

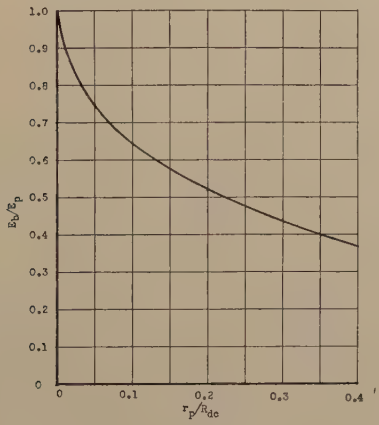

Fig. 3

The above equation is shown plotted in Fig. 3. The net voltage causing maximum current to flow in the modifier is therefore $E_p - E_b$ and the maximum anode current is

$$i_{\max} = \frac{E_p - E_b}{r_p} = \frac{E_b}{r_p}\left(\frac{E_p}{E_b} - 1\right). \tag{28}$$

Thus $i_{\max}$ may be determined once r_p and E_b are known. R_{dc} is needed to determine the ratio of E_p to E_b from Fig. 3. R_{dc} is given approximately by (3) solved for R_{dc} and is

$$R_{dc} = \frac{E_b}{I_{18}} = \frac{E_b^2}{W_c} \tag{29}$$

where,

I_{18} is the modifier direct current.

As a sample calculation, suppose we take the case of a UV848 modifier for the UV863 delivering a 5-kilowatt carrier. Then from (29),

$$R_{dc} = \frac{14000^2}{5000} = 39200 \text{ ohms.} \tag{30}$$

The tube data for the UV848 shows that $r_p = 1800$ ohms approximately. Thus

$$\frac{r_p}{R_{dc}} = \frac{1800}{39200} = 0.046. \tag{31}$$

From Fig. 3 for $r_p/R_{dc} = 0.046$ we read off

$$\frac{E_b}{E_p} = 0.765. \tag{32}$$

Since $E_b = 14000$, from (32),

$$E_p = \frac{14000}{0.765} = 18300 \text{ volts.} \tag{33}$$

The peak plate current through the UV848 is thus, from (28),

$$i_{\max} = \frac{18300 - 14000}{1800} = \frac{4430}{1800} = 2.46 \text{ amperes.} \tag{34}$$

This tube is capable of carrying a peak current of 9.5 amperes with 22 volts on the filament; it is therefore larger than necessary and the filament voltage may be reduced to about 19.5 volts in actual operation for this yields an emission of 3.5 amperes and is therefore still more than adequate. The tube life will be increased to four times normal by this filament-voltage reduction.

We are next concerned with the anode loss in the modifier. In Appendix II it is shown that the loss is

$$W_L = \frac{E_p^2}{r_p}\left[\frac{\left(\frac{\pi}{2} - \beta\right)\left(1 + 2\frac{E_b^2}{E_p^2}\right) + \frac{\sin 2\beta}{2} - \frac{4E_b}{E_p}\cos\beta}{2\pi}\right]$$

$$= \frac{E_p^2}{r_p}[N]. \tag{35}$$

The factor N is a function of E_b/E_p and is shown plotted in Fig. 4. For the UV848 just cited, $E_b/E_p=0.765$, and from Fig. 4 the factor N is thus 0.007. The anode loss is therefore

$$W_L = \frac{18300^2}{1800}(0.007) = 1300 \text{ watts.} \tag{36}$$

The tube is rated at 5 kilowatts dissipation so that it is well within rated limits on dissipation. This loss is not returnable in the form of

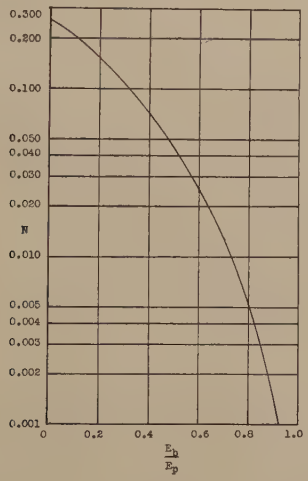

Fig. 4

direct current and will affect the over-all system efficiency by requiring the direct current fed back plus the losses to equal the carrier power so that the actual dissipation will be approximately

$$W_L = \frac{1300}{5000 + 1300}[5000] = 1030 \text{ watts.} \tag{37}$$

The direct current fed back will be

$$W_{dc} = 5000 - 1030 = 3970 \text{ watts.} \tag{38}$$

The over-all system efficiency will thus be

$$\text{eff.} = \frac{5000}{13400 - 3970} = 53\%. \tag{39}$$

A more exact treatment of this subject is given in Appendix III. This figure can be increased by using a modifier of lower internal re-

sistance or by using a modifier circuit which suppresses harmonic-current flow so that the peak current is reduced and the current flows through the anode in essentially rectangular blocks.

Variations of Basic Circuit

The basic circuit illustrated in Fig. 1 may be varied in a number of ways. For instance, the quarter-wavelength section may be of the type wherein the shunt elements are inductors and the series element is a capacitor. It is also possible to take the modifier out of the anode circuit of the power amplifier completely and operate it through a circuit coupled to the main tank.

A circuit in which the quarter-wave section is entirely dispensed with is shown in Fig. 5. In this circuit the modifier is connected in shunt

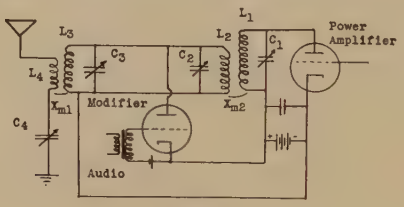

Fig. 5

with the load circuit. The circuit is arranged as follows: The tank L_1C_1 connected in the anode circuit of the power amplifier is tuned to resonance. Coupled to L_1 is the secondary inductor L_2 whose reactance is tuned out by shunt capacitor C_2. In parallel with L_2 is the shunt-tuned circuit L_3C_3 which is likewise tuned to resonance. Coupled to L_3 is the antenna inductor L_4. The antenna circuit is tuned to resonance by means of the combination of antenna reactance and C_4. This, then, is really a tuned link-coupled circuit arrangement. The modifier is connected in shunt across the link circuit and the rectified radio frequency in the form of direct current is fed back into the plate supply for the power amplifier as in the basic circuit of Fig. 1.

In operation, with the modifier biased beyond cutoff, the antenna current becomes twice that under carrier conditions; and under carrier conditions, when the modifier bias is normal, the effective resistance of the modifier equals that coupled in from the antenna circuit so that the output of the tube is divided equally between those two loads. Modulation of the antenna current is then made possible by simply varying the modifier-grid potential at an audio-frequency rate.

The analysis of this circuit is as follows: If R_a is the antenna resistance, then the resistance coupled into L_3 is given by the coupled-circuit equation

$$R_1 = \frac{X_{m1}^2}{R_a} \tag{40}$$

where X_{m1} = mutual reactance between L_3 and L_4; R_1 in turn appears as a shunt resistance across L_3 of the value

$$R_2 = \frac{X_3^2}{R_1} = \frac{X_3^2 R_a}{X_{m1}^2} \tag{41}$$

where X_3 is the reactance of L_3.

Now the modifier effective resistance is also connected in shunt with L_3, and if this is called R_3, then the total effective resistance across L_3 is given by

$$R_4 = \frac{R_2 R_3}{R_2 + R_3} = \frac{X_3^2 R_a R_3}{X_{m1}^2 \left(R_3 + \frac{X_3^2 R_a}{X_{m1}^2} \right)} . \tag{42}$$

The shunt resistance R_4 appears as a series resistance in series with L_2 of the value

$$R_5 = \frac{X_2^2}{R_4} = \frac{X_2^2 (R_3 X_{m1}^2 + X_3^2 R_a)}{X_3^2 R_a R_3} \tag{43}$$

where X_2 = reactance of L_2.

By coupled-circuit theory this in turn appears as a series resistance in series with L_1 of

$$R_6 = \frac{X_{m2}^2}{R_5} = \frac{X_{m2}^2 X_3^2 R_a R_3}{X_2^2 (R_3 X_{m1}^2 + R_a X_3^2)} \tag{44}$$

where X_{m2} = mutual reactance between L_1 and L_2.

Finally, this resistance appears as a shunt resistance across L_1 of

$$R_7 = \frac{X_1^2}{R_6} = \frac{X_1^2 X_2^2 (R_3 X_{m1}^2 + R_a X_3^2)}{X_{m2}^2 X_3^2 R_a R_3} . \tag{45}$$

This then is the load resistance which the power amplifier sees. When the modulation is 100 per cent positive it is known that $R_3 = \infty$. Substituting this in (45), there is obtained for the tube load

$$R_7 = \frac{X_1^2 X_2^2 X_{m1}^2}{X_{m2}^2 X_3^2 R_a} . \tag{46}$$

The tube is now delivering four times normal carrier power. Under carrier conditions and no modulation, it has been said that the antenna and modifier loads were equal, or that $R_3 = R_2$. From (41), then,

$$R_3 = \frac{X_3^2 R_a}{X_{m1}^2} \cdot \tag{47}$$

Substituting (47) in (45), the load upon the power amplifier is found to become

$$R_7 = \frac{2X_1^2 X_2^2 X_{m1}^2}{X_{m2}^2 X_3^2 R_a} \cdot \tag{48}$$

This is seen to be twice the resistance of (46). Hence, with the power amplifier having full grid excitation, the tube output will be one half that under the 100 per cent positive-modulation condition, and since only half of this power gets to the antenna, the antenna power is one fourth, or its current one half that of 100 per cent positive modulation, which is the correct condition for good modulation.

No attempt will be made in this paper to specify the reactances and coupling coefficients since a large number of combinations will give the proper operating conditions. It may be suggested as a guide, however, that X_2 and X_3 have about equal reactance and that each reactance be about one eighth the impedance of R_3. The value of R_3 is identical to that of (3) or

$$R_3 = \frac{E_b^2}{2W_c} \cdot \tag{49}$$

The capacitors C_2 and C_3 may be lumped into a common capacitor for economy if desired and the capacitance of the modifier anode circuit allowed for in designing the composite capacitor. This unit will be the tuning unit for the link circuit.

The adjustments of this circuit will then be only two: those of providing the correct mutual reactances X_{m1} and X_{m2}. These may be quickly made in practice to give correct operating conditions.

This variation of the system is a simple one to set up and adjust, which may make it adaptable to amateur radio as well as to broadcast and other services.

Test Transmitter

A transmitter was constructed to demonstrate the system on a small scale. The basic circuit of Fig. 1 was utilized. The tube complement consisted of a 41 radio-frequency oscillator, on 660 kilocycles plate modulated by a 6A5G with a 6L6G power amplifier. The modifier was a 2A3 and the load was a 1V rectifier serving as a dummy antenna. The direct-current power dissipated in the 1V load resistor corresponded to the usual antenna power radiated. The 6A5G was fed from a 6F6G speech amplifier.

With 250 volts for E_b, the plate current of the power amplifier was 23 milliamperes, and under no-modulation conditions the modifier direct current was 7 milliamperes. The dummy-antenna power was 2.05 watts. Thus the power-supply drain was 250 $(23-7)$ $(10^{-3})=4.0$ watts, yielding a conversion efficiency of 2.05/4.0 or 51 per cent. This agrees closely with the calculated value for this tube when E_2 is as low as 250 volts.

It should be noted that inverse feedback may be employed to reduce distortion. This was provided for in the test transmitter by feeding audio frequency from the dummy antenna back to the speech-amplifier circuit. Very little feedback was obtained because of the low amplification factors of the tubes used; however, the over-all distortion of the transmitter measured at 400 cycles as shown in Fig. 9 indicates satisfactorily low distortion for many applications.

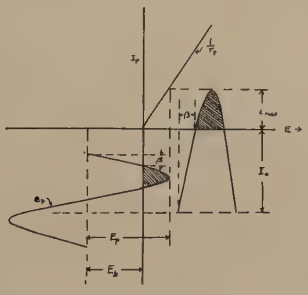

Fig. 6

Conclusion

A novel yet simple high-efficiency modulation system has been described which should be of greatest importance for high-power modulated radio-frequency apparatus where a power saving can be proved in. As a consequence, its field would seem to lie in continuous-duty equipment rather than in the field of intermittent-duty equipment.

Appendix I

Determination of the Relation between E_b/E_p and r_p/R_{dc}

If the rectification characteristic of the modifier as shown in Fig. 6 is considered, it will be seen that the average anode current will be

$$I_{dc}=\frac{1}{2\pi}\int_{\beta}^{\pi-\beta}(i_{\max}+I_0)\sin\phi\,d\phi-\frac{I_0(\pi-2\beta)}{2\pi} \qquad (50)$$

where,

$i_{\max}$ = peak current through the tube

I_0 = distance from cutoff to the fictitious alternating-current axis of sinusoidal current waves, measured as current

β = angle between zero on the fictitious axis to the point at which current just begins to flow.

It is likewise seen that the maximum current through the tube is determined by

$$i_{\max} = \frac{E_p - E_b}{r_p} \tag{51}$$

where,

E_b = direct voltage developed which is equivalent to a negative bias

E_p = peak of alternating-voltage swing measured from the fictitious alternating-voltage axis

r_p = tube-plate resistance.

The fictitious current I_0 is determined by

$$I_0 = \frac{E_b}{r_p} . \tag{52}$$

Substituting (51) and (52) in (50),

$$I_{dc} = \frac{E_p}{2\pi r_p} \int_{\beta}^{\pi-\beta} \sin \phi \, d\phi - \frac{E_b(\pi - 2\beta)}{2\pi r_p} . \tag{53}$$

Now

$$I_{dc} = \frac{E_b}{R_{dc}} \tag{54}$$

where R_{dc} = equivalent direct-current load resistance.

Substituting (54) in (53) and solving for r_p/R_{dc},

$$\frac{r_p}{R_{dc}} = \frac{E_p}{2\pi E_b} \int_{\beta}^{\pi-\beta} \sin \phi \, d\phi - \frac{\pi - 2\beta}{2\pi} . \tag{55}$$

Performing the integration and substituting in the limits, remembering that $\beta = \sin^{-1} E_b/E_p$, (55) becomes

$$\frac{r_p}{R_{dc}} = \frac{\cot \beta - \left(\frac{\pi}{2} - \beta\right)}{\pi} . \tag{56}$$

Fig. 3 was calculated from (56).

Appendix II

Determination of Loss on the Modifier Anode

The watts loss on the modifier anode is the integral of the product of current through the tube and the voltage drop across the tube for the period of current flow. If e_p is the instantaneous radio-frequency voltage, then the instantaneous current through the tube is

$$i_p = \frac{e_p - E_b}{r_p} . \tag{57}$$

The voltage drop across the tube is

$$e_t = e_p - E_b. \tag{58}$$

The watts loss is then

$$W_L = \frac{1}{2\pi}\int_{\beta}^{\pi-\beta} i_p e_t d\phi = \frac{1}{2\pi r_p}\int_{\beta}^{\pi-\beta} (e_p - E_b)^2 d\phi. \tag{59}$$

The equation for e_p from Fig. 6 is seen to be

$$e_p = E_p \sin \phi. \tag{60}$$

Substituting (60) in (59) and performing the indicated integration and remembering $\beta = \sin^{-1} E_b/E_p$, there results for the watts loss

$$W_L = \frac{E_p^2}{r_p}\left[\left(\frac{1}{4} - \frac{\beta}{2\pi}\right)\left(1 + \frac{2E_b^2}{E_p^2}\right) + \frac{\sin 2\beta}{4\pi} - \frac{2E_b \cos \beta}{\pi E_p}\right] = \frac{E_p^2 N}{r_p} \tag{61}$$

where N is the part in the brackets. N is shown plotted as a function of E_b/E_p in Fig. 4.

Appendix III

Determination of E_b/E_p from Known Data

In order to determine the watts loss it is necessary to know E_b/E_p so that E_p and N may be found for use in (61). Usually the data known are the direct supply voltage E_b, the modifier plate resistance r_p, and the carrier power W_c. With these data known it is possible to determine E_b/E_p in the following way:

The total power in the modifier circuit consists of two parts: the anode loss of the modifier and the power delivered as direct-current to the power supply. This total power is equal to the carrier power, or,

$$W_c = \frac{E_p^2 N}{r_p} + \frac{E_b^2}{R_{dc}} \tag{62}$$

where R_{dc} is the equivalent load on the modifier.

Solving (62) for R_{dc} and (56) for R_{dc} there result the following simultaneous equations:

$$R_{dc} = \frac{E_b^2}{W_c - \frac{E_p^2 N}{r_p}} \qquad (63)$$

$$R_{dc} = \frac{r_p \pi}{\cot \beta - \left(\frac{\pi}{2} - \beta\right)} \qquad (64)$$

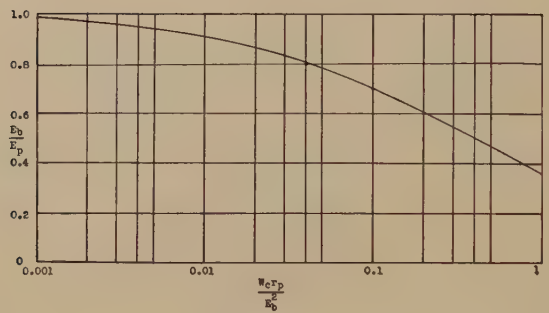

Fig. 7

Eliminating R_{dc} from (57) and (58) by equating right-hand members, the following may be solved for in the resultant equation:

$$\frac{W_c r_p}{E_b^2} = N\left(\frac{E_p}{E_b}\right)^2 + \frac{\cot \beta - \left(\frac{\pi}{2} - \beta\right)}{\pi}. \qquad (65)$$

The entire right-hand member is a function of only one variable, namely, E_b/E_p, for $\beta = \sin^{-1} E_b/E_p$, and the left-hand term is made up of the known data; hence, for any given left-hand member a definite E_b/E_p exists. Fig. 7 is a graphical solution of (65). After E_b/E_p has been determined it may be used to obtain E_p, N, W_L, R_{dc}, and the power fed back into the direct-current supply.

Appendix IV

Determination of Effective Alternating-Current Resistance of Modifier Circuit

It has been stated that the alternating-current resistance of the modifier circuit is approximately equal to half the direct-current load

resistance. This is true only where $E_b = E_p$. As E_p gradually increases above E_b, the alternating-current resistance increases as a function of

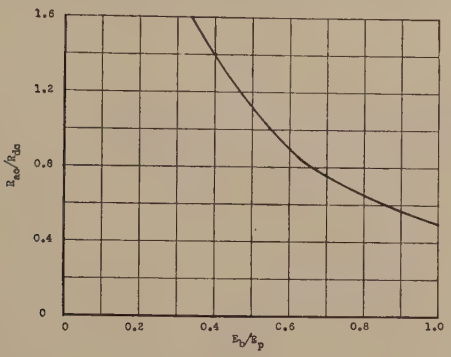

Fig. 8

the direct-current resistance until it becomes infinitely greater as E_b becomes zero. The way in which this varies is of use in determining the surge impedance of the quarter-wavelength section and its circuit constants. The alternating-current resistance may be determined from

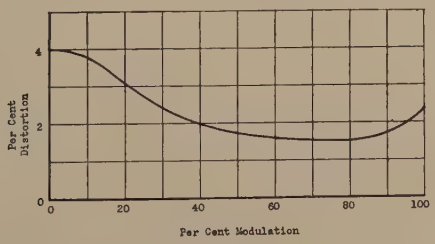

Fig. 9

the total modifier power, equation (62), where the carrier power is expressed as a function of R_{ac}, thus,

$$\frac{E_p^2}{2R_{ac}} = \frac{E_p^2 N}{r_p} + \frac{E_b^2}{R_{dc}}. \tag{66}$$

From (56) it is seen that

$$r_p = \frac{R_{dc}\left[\cot\beta - \left(\frac{\pi}{2} - \beta\right)\right]}{\pi}. \tag{67}$$

Substituting (67) in (66) for r_p, and solving for R_{ac}/R_{dc},

$$\frac{R_{ac}}{R_{dc}} = \frac{1}{2\left[\dfrac{E_b^2}{E_p^2} + \dfrac{N\pi}{\cot\beta - \left(\dfrac{\pi}{2} - \beta\right)}\right]}. \tag{68}$$

Thus it is possible to solve for the ratio of R_{ac} to R_{dc} in terms of E_b/E_p. The latter quantity is found from the results of Appendix III from certain known data in the transmitter. Fig. 8 shows how R_{ac}/R_{dc} varies as a function of E_b/E_p.

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A PHASE-OPPOSITION SYSTEM OF AMPLITUDE MODULATION*

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Summary—*The increasing economic importance of power efficiency for high-power broadcast stations is illustrated in Fig. 1 showing that some 50 per cent saving is possible. Different known modulation schemes are compared as regards necessary input power, including the recent Chireix and Doherty systems. The influence of rates for electric power is stressed. A so-called "phase-opposition" system is discussed starting with simple circuit theory, first for series- and then for parallel-tuned circuits. Principally the system is based on the co-operation of two generators; i.e., one "carrier" generator constantly excited, and one "side-band" generator whose excitation is changed 180 degrees in phase according to the phase of modulation. Mean input power is calculated for this system, giving about the same input as good systems for about 30 per cent modulation, but 10 to 25 per cent more power at 100 per cent modulation. The paper is concluded with a consideration of tube characteristics and possible means for obtaining linearity are shown.*

I. Introduction

ALTHOUGH not the only possible method of modulation, the system which uses pure amplitude modulation of a transmitter, has become by far the most dominant system. As is well known, this system is now well developed, and we cannot possibly expect much more to be gained, either as to the degree of distortion or as to degree of modulation obtained. The reason why pure amplitude modulation has become almost universally adopted, is, in the opinion of the writer, the relatively simple construction and operation of a receiver which will amplify and detect such modulated signals. Other modulating systems may not be more complicated on the transmitting side, in fact some are even simpler, but as a rule those systems require far more complicated receivers to obtain equal results. At the present time, therefore, one may state that every transmitter which is tuned in on numerous receivers will be amplitude modulated, and other systems of modulation are to be found in transmitters tuned in on few receivers only.

As is well known, the anode-voltage-control system (whether by series or parallel control) accredited to R. A. Heising[1] has become the dominant method for amplitude modulation mainly due to the fact

* Decimal classification: R148×R355.8. Original manuscript received by the Institute, December 24, 1937.

[1] R. A. Heising, "Modulation in radio telephony," Proc. I.R.E., vol. 9, pp. 305–352; (August), (1921).

that high degrees of modulation easily are obtained with small amounts of distortion with this system.

The anode-voltage-modulating system, although originally invented and used with triodes, may be used also for pentodes equally well, as later investigations showed.

Methods using grid-control, either the "grid-current" or the "grid-voltage" system, also are in use to some extent. They are rather limited, however, because of the difficulty of obtaining 100 per cent modulation with tolerable distortion, when using this system. Recent measurements[2] on transmitting pentodes have, however, shown the possibility of obtaining amplitude modulation of a quality comparable to the "Heising modulation" by adopting a pure voltage control in the supressor-grid circuit of a pentode.

When the demand for higher carrier powers arose, the so-called "low-power-modulation" system was adopted, where the modulation takes place at a comparatively low energy level with subsequent amplification in linear high-frequency class B amplifiers, working with a low anode efficiency of about 33 per cent.

It is interesting now to recapitulate some of the arguments which led to this system in preference to the "high-power" system, where modulation takes place in the last stage of the transmitter.

The arguments were mainly as follows:

(1) When using the "high-power" system, the last high-frequency stage could be set to work with a reasonably high anode efficiency of some 60 to 70 per cent, but the output stage of the modulator had to work as a class A stage with a very poor anode efficiency of 30 per cent in order to get "linear amplification" and tolerable distortion. Based on this last assumption we could quite easily calculate the losses of a "high-power" system to be higher than those of a "low-power" system with the same carrier power.

(2) The construction of a high-power modulator using iron-core transformers of high power for a considerable frequency range was thought to present prohibitive difficulties with regard to the allowable tolerances for distortion and frequency response. (Iron distortion.)

As later developments have shown, both arguments were based on rather weak assumptions. It is a fact, however, that the low-power or low-level modulation system generally was accepted as the most suitable system, and consequently came in use on most broadcast transmitters erected. One may more or less regard this as the adopted

[2] C. J. de Lussanet de la Sabloniere, "The new transmitting pentode P.C. 1,5/100," *Philips Transmitting News*, vol. 1, no. 3; December, (1934).

standard system for broadcast transmitters of a carrier energy up to say 120 to 150 kilowatts.

The listeners' demand for increased quality of received programs, together with the ever-increasing number of broadcast transmitters erected, logically led to higher carrier powers, and the general trend of actual development still seems to follow this line.

With increasing power, the part of the total running costs which are represented by the cost of electric power drawn from the mains, becomes the more dominant part of the total costs. Thus, in recent years the power efficiency of the modulation system has been taken up for investigation and several energy-saving systems of modulation have been developed and put into practical use in different countries.

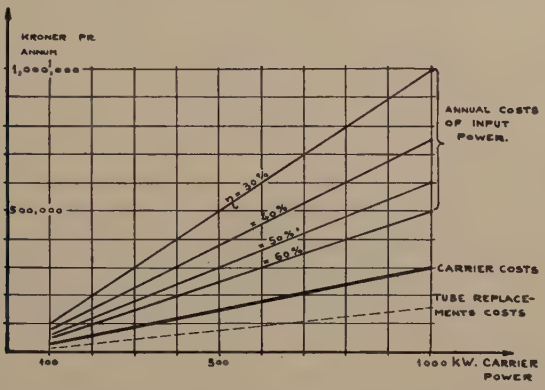

Fig. 1—Annual power costs for carrier energies from 100 to 1000 kilowatts, upper four curves give costs for anode input power with respectively 30, 40, 50, and 60 per cent efficiency, then follows carrier energy costs alone, and below the annual tube-replacement costs. For underlying assumptions refer to the text.

The increasing importance of the efficiency of modulating systems is illustrated in Fig. 1. where the abscissas represent carrier power in kilowatts and the ordinates are annual power costs for the power input to the anodes of the output stage alone.

The calculation of the data shown in Fig. 1 is based on the following assumptions:

Time of operation: 6000 hours per year.

Rate for electric power: kroner 0.05 per kilowatt-hour, independent of the number of kilowatts taken from the mains.

The four upper curves give annual costs (in Norwegian kroner) for the input power with efficiencies of respective $\eta = 30$, 40, 50, and 60 per cent. The next curve shows annual costs of the carrier power,

ordinates taken from this line up to one of the upper curves thus representing the costs of lost power.

For comparison a curve marked "tube-replacements costs" is drawn in Fig. 1. The data for this last curve are as follows:
Price per 200-kilowatt water-cooled tube: kroner 8000.
Average useful lifetime of tubes: 1 year of 6000 operating hours.

The curves of Fig. 1 are self-explanatory, so we see that an increase from 30 to 60 per cent efficiency for a 1000-kilowatt transmitter saves an annual expense of kroner 500,000 in the running costs.

Generally the losses of a transmitter are dissipated in cooling plants, the erecting costs of which naturally increase with increasing losses. Thus we find that increasing the efficiency also reduces the erecting expenses for the whole transmitter. So far as the writer is informed regenerating of cooling-water energy, for heating purposes or other aims, is not practiced to any extent of importance.

It should be admitted that the base of Fig. 1 is a fixed kilowatt-hour price of power. If the kilowatt-hour price is dependent on the load in kilowatts however, similar curves (then not straight lines) could always be set up.

In some cases the terms of power payment are influenced by the peak power taken from the mains, so that the efficiency must be considered in close connection with the properties of the modulating systems as to how input power varies with degree of modulation. The most economical system would then combine a high efficiency with lowest possible peak power. Individual circumstances must in such cases be considered in detail.

II. Comparison Between Input Powers of Known Modulation Systems

It is thought useful to give a brief comparative description of some existing modulation systems as to the necessary input power and its dependence on the degree of modulation. The basis of this comparison will be an unmodulated carrier energy of 100 kilowatts which we want to modulate using different systems, and we consider only the input power to the last stage, for high-power systems including the input to the last modulator stage.

The following notations will be used:

W_c = carrier power unmodulated
W_i = anode-input power
η_o = anode efficiency of high-frequency amplifier
η_m = anode efficiency of last modulator amplifier
m = degree of modulation.

A. High-level system, modulator operated as a class A stage

The total anode-input power to the high-frequency amplifier and modulator is, as simple considerations show,

$$W_i = \frac{W_c}{\eta_\sigma}\left\{1 + \frac{m^2}{2\eta_m}\right\}. \qquad (1)$$

As for class A operation of the modulator the efficiency is proportional to the square of the output voltage, η_m is proportional to m^2, and consequently the total input is constant, thus independent of modulation.

As to actual obtainable figures, $\eta_\sigma = 0.7$ seems a reasonable value to be realized at least for not too high frequencies. For the modulator $\eta_m = 0.3$ may be accepted for $m = 100$ and tolerable distortion. This gives for $W_c = 100$ kilowatts. Thus,

(a) $$W_i = \frac{100}{0.7}\left(1 + \frac{1}{0.6}\right) = 382 \text{ kilowatts, constant.}$$

B. Low-level system, premodulation

The high-frequency amplifier is operated as a class B stage and excited with premodulated voltages, the anode efficiency is proportional to input voltage, the mean anode efficiency is thus independent of modulation, and in practice equal to 33 per cent.

(b) $$W_i = \frac{W_c}{\eta_\sigma} = \frac{100}{0.33} = 303 \text{ kilowatts, constant.}$$

C. High-level system, modulators operated as class B stage

Formula (1) also applies to class B operation of modulators. We only have to substitute practical values for η_m to calculate W_i. The theoretical maximum value[3,4] of η_m is $\pi/4 = 0.785$. There is then assumed an anode alternating voltage of amplitude equal to the anode direct voltage, and complete cutoff, i.e. anode rest current equal to zero. For these theoretical assumptions η_m would be proportional to m, and consequently W_i would rise from W_c/η_σ to a maximum value in linear relation to m. Both assumptions do not hold in practice; as is well known, the anode voltage is limited to smaller values and the rest current is actually about 30 per cent of the maximum value at $m = 100$. In practice the actual maximum value for η_m seems to lie around $\eta_m = 0.5$.

[3] I. E. Mouromtseff and H. N. Kozanowski, "Comparative analysis of water-cooled tubes as class B audio amplifiers," Proc. I.R.E., vol. 23, pp. 1224–1251; October, (1935).

[4] Chambers, Jones, Fyler, Williamson, Leach and Hutcheson, "The WLW 500-kilowatt broadcast transmitter," Proc. I.R.E., vol. 22, pp. 1151–1180; October, (1934).

Assuming the modulator input to rise linearly with m from 0.3 to the maximum value given by $\eta_m = 0.50$, one finds

$$W_i = \frac{W_c}{\eta_\sigma}\left\{1 + \frac{0.3 + 0.7m}{2 \cdot \eta_{\max}}\right\}. \tag{2}$$

With the above accepted value $\eta_\sigma = 0.7$ and $\eta_{\max} = 0.5$ we have for $W_c = 100$ kilowatts

$$W_i = 186 + 100m \text{ kilowatts.} \tag{c}$$

D. The Chireix system

The Chireix system[5] called by him "outphasing modulation" divides the high-frequency output power equally between two high-frequency amplifiers, both working in class C operation and excited with grid voltages which are modulated in relative phase in such a way that at zero amplitude the two voltages are in phase opposition; at maximum amplitude the two voltages are in phase. The principle is of course to keep both tubes in class C operation with high efficiency for all load amplitudes. If theoretically this could be secured, the efficiency is a constant and the input power would depend on the degree of modulation (easily verified), as given by

$$W_i = \frac{W_c}{\eta_\sigma}\left(1 + \frac{m^2}{2}\right). \tag{3}$$

This equation simply says that the input power is rising in the same proportion as the load power due to the amplitude modulation. Chireix[5] has shown, however, that for small load amplitudes the working impedances for the two tubes are no longer purely resistive and the efficiency is thus considerable lower, with the consequence that for higher values of m the input power is higher than given by (3). For a carrier power of $W_c = 100$ kilowatts the following input figures are stated by Chireix for different modulation degrees:

(d)

$m =$	0	0.3	0.6	0.8	1.0
$W_i =$	167	174	212	246	294 kilowatts

These figures are based on a tube efficiency of 75 per cent and two circuits each of 90 per cent efficiency, giving an over-all efficiency of $\eta_\sigma = 60$ per cent as stated by Chireix.

[5] H. Chireix, "High power outphasing modulation," Proc. I.R.E., vol. 23, pp. 1370–1392; November, (1935).

E. The Doherty System

This system[6] also divides the output power between two tubes but this division is equal between the two tubes only for full-load amplitude. For amplitudes up to the carrier value only tube 1 delivers energy; tube 2 gives energy only for amplitudes greater than the carrier value.

Both tubes are excited with premodulated high-frequency voltages which are 90 per cent dephased due to the impedance-inverting network inserted between the two tubes. The "carrier," tube 1, is (and must be) class B operated; the "top" tube 2 is biased to give zero anode current with carrier grid input, and consequently works with a little higher efficiency than tube 1 (when operating).

According to measurements made by Doherty[6] (see his Figs. 5 and 11) the input, direct current to tube 1 may be represented, for ideal linear conditions, by

$$i_{a1} = 1 + m \sin x \tag{4}$$

where the carrier current is set equal to 1.

The input direct current to tube 2 has then the value

$$i_{a2} = 2m \sin x \tag{5}$$

for $\pi > x > 0$ and $i_{a2} = 0$ for $2\pi > x > \pi$, also under the assumption of linearity.

The mean values over a complete modulation cycle of the two currents are, respectively,

$$\text{for} \quad i_{a1} \quad : \quad 1$$

$$\text{for} \quad i_{a2} \quad : \quad 2m/\pi.$$

Whence follows the input power at a modulation degree m (assuming a constant plate voltage)

$$W_i = \frac{W_c}{\eta_\sigma}\left\{1 + \frac{2}{\pi} \cdot m\right\}. \tag{6}$$

Assuming an over-all efficiency of about 60 per cent for the carrier according to Doherty we get for a 100-kilowatt carrier

$$W_i = 167 + 106 \cdot m \text{ kilowatts.} \tag{e1}$$

This is not quite in accordance with the statement of Doherty, who deduces the $1 + m^2/2$ law for the input rise during modulation. It should

[6] W. H. Doherty, "A new high efficiency power amplifier for modulated waves," Proc. I.R.E., vol. 24, pp. 1163–1182; September, (1936).

be noted that the above calculations give too small mean values for higher modulation degrees, as the currents actually do not go down to zero for carrier excitation on tube 2 nor for zero excitation of tube 1.

The assumption of a constant efficiency during modulation leads to the following input for a 100-kilowatt stage:

(e₂) $$W_i = 167 + 83.5 \cdot m^2 \text{ kilowatts.}$$

In the opinion of the writer this second-power law necessarily requires that i_{a2} follows a pure second-power law with excitation if we consider i_{a1} to be linear with excitation.

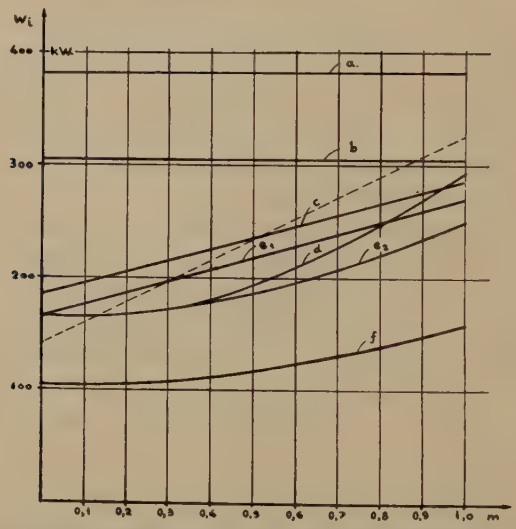

Fig. 2—Anode input power to the last stage of a 100-kilowatt carrier-energy transmitter, using different systems of modulation. Curve *a* is for high-power modulation with a class A modulator, *b* the low-level system, *c* the high-power system with class B modulators, *d* the Chireix system, e_1 the Doherty system assuming "linear" currents, e_2 the Doherty system assuming a constant efficiency, and *f* a hypothetical "ideal" system with a constant efficiency of 95 per cent.

Concluding this comparison we should like to state that an ideal modulation system would require a constant efficiency of say 95 per cent as is obtained in other branches of the electrotechnical field. Such an ideal system would have an input power of

(f) $$W_i = 105 + 52.5 \cdot m^2 \text{ kilowatts.}$$

for 100-kilowatt carrier energy.

The result of the preceding comparative considerations is graphically represented in Fig. 2, where the mean anode input power is

plotted as a function of the degree of modulation. All curves are evaluated for 100-kilowatt unmodulated carrier power in the load (antenna). In practice the two figures of importance are the following:

(1) Input power at 30 per cent modulation.
(2) Peak input power at 100 per cent modulation, for which the power supply must be rated.

These two figures together with the assumed settings and efficiencies of tubes are tabulated below:

TABLE I

No.	System	Modulators		High-frequency tubes		Input power kilowatts	
		Class	η max.	Class	η max.	$m=30\%$	$m=100\%$
(a)	High-level	A	30%	C	70%	382	382
(b)	Low-level	A	30%	B	66%	303	303
(c)	High-level	B	50%	C	70%	216	286
(d)	Chireix	—	—	C	60%	174	294
(e_1)	Doherty	—	—	B	60%	198.8	273
(e_2)	Doherty	—	—	B	60%	174.5	250.5
(f)	"Ideal"	—	—	C	95%	109.7	157.5

In this table (e_1) refers to the linear-current assumption mentioned above, (e_2) to an assumed constant efficiency of 60 per cent.

From these evaluations the following conclusions seem reasonable:

(1) There is still a 60 per cent saving to be had by raising the efficiency of high-frequency generators to 95 per cent.
(2) If electric power is paid according to peaks alone, there is no real important difference between systems (b), (c), (d), and (e). Those four systems also require about the same rating of power-supply units. Systems (c) and (a) however require extra tube expenses due to the high-level modulator.
(3) If electric power is paid at a constant kilowatt-hour rate, systems (d) and (e) are the most favorable ones.

The above comparative deductions refer only to the necessary input power of the last stage; in judging between modulation systems other factors have also to be taken into consideration, among which the following should be mentioned:

(a) Total input power including preceding stages
(b) Total tube expenses
(c) Degree of distortion at 100 per cent modulation
(d) Frequency response
(e) Operation and maintenance simplicity.

Following this discussion of existing modulation systems known to the writer, we proceed with a discussion of another possible system, of amplitude modulation called here "a phase-opposition system."

III. Simple Circuit Theory of Phase Opposition

We start our considerations by a general discussion of the simple series networks pictured in Fig. 3.

Here a primary series circuit, R_1, C_1, L_1, carrying the current I_1 is connected to a generator G_1, which deliveries the voltage E_1 at its terminals.

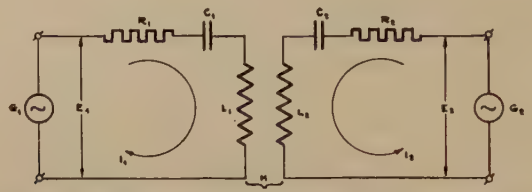

Fig. 3—Two simple series networks inductively coupled (M) and connected to generators G_1 and G_2, respectively.

Inductively coupled to the primary circuit is a secondary circuit R_2, C_2, L_2 carrying the current I_2 and connected at its terminals to a second generator G_2 delivering a voltage E_2 of the same frequency as E_1. We assume circuits to be series tuned, or

$$\omega^2 = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}, \tag{7}$$

for which case the equations of the system simplify to

$$\begin{aligned} E_1 &= I_1 R_1 + j\omega M I_2 \\ E_2 &= I_2 R_2 + j\omega M I_1. \end{aligned} \tag{8}$$

These equations give for the two currents I_1 and I_2 the following expressions:

$$I_1 = \frac{E_1 - \dfrac{j\omega M}{R_2} \cdot E_2}{R_1 + \dfrac{\omega^2 M^2}{R_2}}, \qquad I_2 = \frac{E_2 - \dfrac{j\omega M}{R_1} \cdot E_1}{R_2 + \dfrac{\omega^2 M^2}{R_1}}. \tag{9}$$

We suppose now the generator G_1 to supply the carrier energy and regard the voltage E_1 as a constant. Generator G_2 supplies the side bands (without carrier) and we shall discuss how currents I_1 and I_2 depend on the voltage E_2 alone. From (9) we immediately see that E_1 and E_2 must have a relative 90-degree phase difference if we want the currents to obtain zero values, necessary for 100 per cent modulation. We consider now the following three typical cases:

A. Carrier condition given by $E_2=0$.

Substituting in (9) gives the two carrier currents

$$I_{1.0} = \frac{E_1}{R_1 + \frac{\omega^2 M^2}{R_2}}, \qquad I_{2.0} = \frac{-\frac{j\omega M}{R_1} \cdot E_1}{R_2 + \frac{\omega^2 M^2}{R_1}} = -\frac{j\omega M}{R_2} \cdot I_{1.0}. \tag{10}$$

The primary current is in phase with the voltage E_1, and directly given by the sum of the primary and introduced secondary resistance. The secondary is 90 degrees dephased with respect to $I_{1.0}$ and simply equal to induced voltage divided by R_2.

B. Zero-amplitude secondary current, $I_2=0$.

To obtain this we must make the secondary voltage equal to

$$E_2 = \frac{j\omega M}{R_1} \cdot E_1. \tag{11}$$

Since $I_2=0$ the primary current is then

$$I_1 = \frac{E_1}{R_1} = I_{1.0}\left(1 + \frac{\omega^2 M^2}{R_1 R_2}\right) \tag{12}$$

which means that E_2 must be just equal to $j\omega M\, I_1$, to cancel the voltage induced in the secondary circuit from the primary current I_1. As remarked above E_2 must be 90 degrees dephased with respect to E_1.

C. Double-carrier-amplitude secondary current, $I_2=2\ I_{2.0}$.

In order to have I_2 doubled, we see from (9) that E_2 must be

$$E_2 = -\frac{j\omega M}{R_1} \cdot E_1. \tag{13}$$

This is exactly the same voltage which gives zero amplitude as shown in (11), but oppositely phased. The primary current for this condition is easily calculated to be

$$I_1 = I_{1.0}\left(1 - \frac{\omega^2 M^2}{R_1 R_2}\right). \tag{14}$$

From (12) and (14) we see that if I_2 varies according to 100 per cent modulation, the corresponding modulation degree of I_1 is

$$\omega^2 M^2 / R_1 R_2.$$

The above results are tabulated below.

TABLE II
E_1 = constant

Condition	E_2	I_2	I_1
A—Carrier	0	$I_{2.0}$	$I_{1.0}$
B—100% down	$\frac{j\omega M}{R_1} \cdot E_1$	0	$I_{1.0}\left(1+\frac{\omega^2 M^2}{R_1 R_2}\right)$
C—100% up	$-\frac{j\omega M}{R_1} \cdot E_1$	$2 \cdot I_{2.0}$	$I_{1.0}\left(1-\frac{\omega^2 M^2}{R_1 R_2}\right)$

From these simple considerations we see that 100 per cent amplitude modulation is possible using the system described. To be noted is the fact that I_1 may change its sign, i.e., phase 180 degrees, for condition C, if $\omega^2 M^2/R_1 R_2 > 1$. It is also noteworthy that the two currents both become modulated to a different degree if $\omega^2 M^2/R_1 R_2 \gtrless 1$ and of opposite phase.

Now we want to calculate the energy supplied to the system by the two generators G_1 and G_2. As both currents are in phase with the respective voltages, the total supplied energy is equal to

$$W = W_1 + W_2 = E_1 I_1 + E_2 I_2. \tag{15}$$

For the carrier condition A, generator G_1 supplies the whole energy alone because then $E_2 = 0$. This carrier energy we denote by $W_{1.0}$.

$$W = W_{1.0} = E_1 \cdot I_{1.0}. \tag{16}$$

Under condition B, i.e., 100 per cent down, generator G_1 also delivers the whole energy because then $I_2 = 0$. We find

$$W = W_1 = W_{1.0}\left(1+\frac{\omega^2 M^2}{R_1 R_2}\right). \tag{17}$$

For condition C, i.e., 100 per cent up, both generators are supplying energy, easily calculated to be

$$W_1 = W_{1.0} \cdot \left(1-\frac{\omega^2 M^2}{R_1 R_2}\right)$$

$$W_2 = W_{1.0} \cdot 2 \cdot \frac{\omega^2 M^2}{R_1 R_2}$$

i.e.,

$$W = W_1 + W_2 = W_{1.0}\left(1+\frac{\omega^2 M^2}{R_1 R_2}\right). \tag{18}$$

Thus we see that the total energy supply is the same as for condition

B even though W_1 may be negative, which means that generator G_1 absorbs energy.

So far we have considered the conditions for complete modulation, we now proceed to general expressions for an arbitrary degree (m) of modulation. By substituting in (9)

$$E_2 = m \cdot \frac{j\omega M}{R_1} \cdot E_1 \cdot \sin x \tag{19}$$

which, as we have seen, corresponds to complete modulation for $m=1$, we find, using the same notations as above

$$\left.\begin{aligned} I_1 &= I_{1.0}\left(1 + \frac{\omega^2 M^2}{R_1 R_2}\, m \sin x\right) \\ I_2 &= I_{2.0}\,(1 - m \sin x) \\ W_1 &= W_{1.0}\left(1 + \frac{\omega^2 M^2}{R_1 R_2}\, m \sin x\right) \\ W_2 &= W_{1.0}\,\frac{\omega^2 M^2}{R_1 R_2}\,(-m \sin x + m^2 \sin^2 x) \end{aligned}\right\} \tag{20}$$

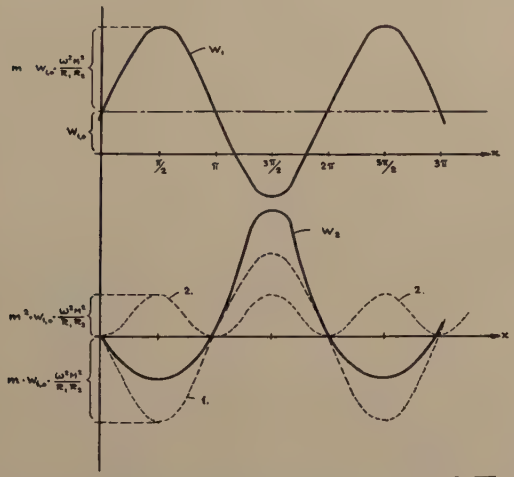

Fig. 4—Showing variations of supplied powers W_1 and W_2, respectively, over the modulation cycle.

In Fig. 4 the variations of the two supplied powers W_1 and W_2 over the modulation cycle are illustrated. W_1 consists of a constant term $W_{1.0}$ and a sine term with amplitude $m \cdot W_{1.0} \cdot \omega^2 M^2 / R_1 R_2$. Thus the maximum power delivered by generator G_1 at 100 per cent modulation is

$$W_{1\max} = W_{1.0}\left(1 + \frac{\omega^2 M^2}{R_1 R_2}\right).$$

W_1 becomes negative for $m=1$ if $\omega^2 M^2/R_1R_2 > 1$, as is easily seen.

W_2 is built up of 3 components, i.e.,

(1) A constant term of magnitude

$$W_{1.0} \cdot \frac{m^2}{2} \cdot \frac{\omega^2 M^2}{R_1 R_2}.$$

(2) A sine term

$$- W_{1.0} \cdot m \cdot \frac{\omega^2 M^2}{R_1 R_2} \sin x.$$

(3) A second-harmonic cosine term

$$- W_{1.0} \frac{m^2}{2} \cdot \frac{\omega^2 M^2}{R_1 R_2} \cdot \cos 2x.$$

As is seen generator 2 must deliver a maximum energy of

$$W_{2\max} = 2 \cdot W_{1.0} \cdot \frac{\omega^2 M^2}{R_1 R_2}.$$

Further it is seen that for $m=1$, W_2 is zero for

$$x = 0, \pi, 2\pi, 3\pi \cdots.$$

and for

$$x = \pi/2, 5\pi/2, \cdots.$$

The total power we find to be equal to

$$W = W_1 + W_2 = W_{1.0}\left(1 + m^2 \frac{\omega^2 M^2}{R_1 R_2} \sin^2 x\right) \qquad (21)$$

as the two sine terms of W_1 and W_2 cancel each other, representing powers which are circulating between the two generators. The maximum of total supplied power is thus just equal to the maximum of generator G_1.

The mean total power over a complete modulation cycle is

$$W_m = W_{1.0}\left(1 + \frac{m^2}{2} \cdot \frac{\omega^2 M^2}{R_1 R_2}\right). \qquad (22)$$

Thus if $\omega^2 M^2/R_1R_2 = 1$, the input power rises in just the same proportion as the modulated energy.

Considering R_2 as the load resistance and R_1 to be the internal resistance of generator G_1, the efficiency of the generator G_1, under carrier conditions is

$$\eta = \frac{\omega^2 M^2}{R_1 R_2 + \omega^2 M^2} . \tag{23}$$

For a carrier energy of 100 kilowatts we write (22) as

$$W_m = \frac{100}{\eta}\left\{1 + \frac{m^2}{2}\cdot\frac{\eta}{1-\eta}\right\}. \tag{24}$$

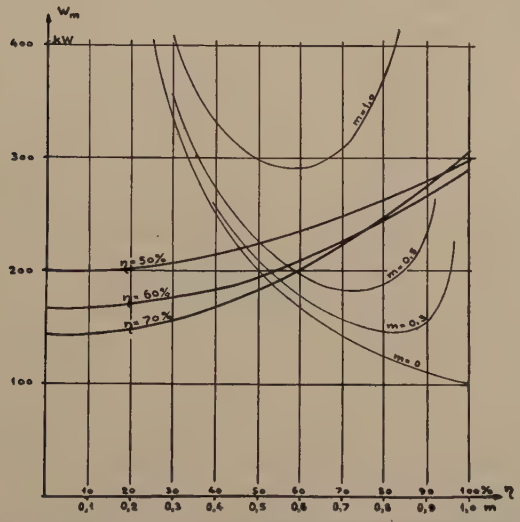

Fig. 5—Supplied total power as a function of the degree of modulation for a 100-kilowatt carrier and 50, 60, and 70 per cent carrier efficiencies, respectively, and the same power as a function of the carrier-efficiency for $m = 0$, 30, 50, and 100 per cent modulation, respectively.

Equation (24) gives the mean total input power as a function of m and η. This relation is graphically illustrated in Fig. 5, where the ordinates represent input power and the abscissas the modulation degree m or efficiency η.

From W_m as a function of m we see that for η between 50 and 70 per cent the peak input power at 100 per cent modulation is about the same while at 30 per cent modulation there is considerable difference, 70 per cent giving least input power.

This is more clearly brought to light by the second family of curves showing W_m as a function of η for definite values of m.

These curves show clearly that for a given value of m there exists

an optimum value of η giving a minimum of input power. These optima for η are

$$\begin{aligned} m &= 1.0 \quad \eta = 0.586 \\ m &= 0.5 \quad \eta = 0.738 \\ m &= 0.3 \quad \eta = 0.829 \\ m &= 0.0 \quad \eta = 1.000 \end{aligned}$$

It is interesting to note that the curve for $m = 1.0$ shows a relative flat minimum just in the region of efficiencies from 50 to 70 per cent which are about the values which present high-frequency amplifiers give. Increasing the carrier efficiency from 70 to 80 per cent reduces to some extent the input power at $m = 0.3$ but increases the peak power at $m = 1.0$ from 310 to 375 kilowatts.

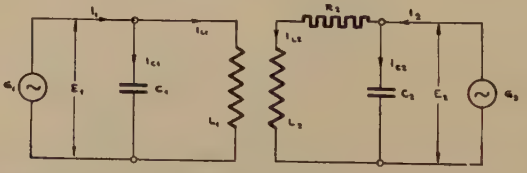

Fig. 6—Simplified diagram of connections for phase-opposition modulation, using parallel-tuned circuits.

The main conclusions we may draw from this discussion seem to be:

(a) Carrier efficiencies of 60 to 70 per cent give the same input at 30 per cent modulation as systems (d) and (e) of Fig. 2.

(b) At full modulation, 300 kilowatts are drawn from the mains against 250 to 295 by systems (e) and (d) Fig. 2.

(c) An increase of carrier efficiency above 70 per cent would reduce input power at normal modulation levels, but increase the peak input power at full modulation.

It will be noted that we have assumed the generator G_2 to have zero internal resistance, for the above deductions. To this very fundamental point we shall have to return later on.

IV. Simple Theory of Phase Opposition Using Parallel-Tuned Circuits

We shall now briefly consider the circuits pictured in Fig. 6, where a generator G_1 with a terminal voltage E_1, is supplying a current I_1 to a primary circuit built up of condenser C_1 and self-inductance L_1 connected in parallel. The currents through condenser C_1 and coil L_1 we designate by I_{c1} and I_{L1}, respectively. A second generator G_2 with a terminal voltage E_2 of the same frequency as E_1 is feeding a current

I_2 into a secondary (load) circuit consisting of a condenser C_2 with current I_{c2} to which is parallel connected a coil having resistance R_2, inductance L_2, and current I_{L2}. Between L_1 and L_2 there is a mutual inductance M.

The equations of this circuit are as follows:

$$\left.\begin{aligned} I_1 &= I_{c1} + I_{L1} \\ I_2 &= I_{c2} + I_{L2} \\ I_{c1} &= E_1 \cdot j\omega C_1 \\ E_1 &= j\omega L_1 \cdot I_{L1} + j\omega M I_{L2} \\ I_{c2} &= E_2 \cdot j\omega C_2 \\ E_2 &= (R_2 + j\omega L_2) I_{L2} + j\omega M I_{L1} \end{aligned}\right\}. \tag{25}$$

We want to study the solutions of these equations from the following point of view:

(1) E_1 is considered constant.

(2) I_2 is considered as the independent variable and we want to know how currents and voltages in the system depend on the supplied current I_2. Especially are we interested in the dependence of the primary current I_1 and the secondary voltage E_2.

From (25) we find the two currents I_2 and I_1 to be, respectively,

$$I_2 = E_2 \cdot j\omega C_2 + \frac{ME_1 - L_1 E_2}{j\omega(M^2 - L_1 L_2) - L_1 R_2}. \tag{26}$$

$$I_1 = E_1 \left(j\omega C_1 + \frac{1}{j\omega L_1} \right) - \frac{M}{L_1} \cdot \frac{ME_1 - L_1 E_2}{j\omega(M^2 - L_1 L_2) - L_1 R_2}. \tag{27}$$

We make the assumption that both circuits are tuned to the same frequency; i.e.,

$$\omega^2 = \frac{1}{L_1 C_1} = \frac{1}{L_2 C_2}. \tag{28}$$

For the reactance of the two coils we write

$$X_1 = \omega L_1, \qquad X_2 = \omega L_2. \tag{29}$$

Further we substitute

$$k^2 = \frac{M^2}{L_1 L_2}, \qquad \delta_2 = \frac{R_2}{X_2} \tag{30}$$

defining, respectively, coupling coefficient and decrement of the secondary circuit.

When we substitute these usual abbreviations in (26) and (28) we have

$$I_2 = \frac{E_2(k^2 + j\delta_2) - k \cdot \sqrt{\frac{X_2}{X_1}} \cdot E_1}{X_2(\delta_2 + j(1 - k^2))} \tag{31}$$

$$I_1 = \frac{k^2 \cdot E_1 - k \cdot \sqrt{\frac{X_1}{X_2}} \cdot E_2}{X_1(\delta_2 + j(1 - k^2))} \cdot \tag{32}$$

From (31) we see that in order to make $I_2 = 0$ the secondary voltage E_2 must be equal to

$$E_{2.0} = E_1 \sqrt{\frac{X_2}{X_1}} \cdot \frac{k}{k^2 + j\delta_2} \cdot \tag{33}$$

This is the secondary voltage which corresponds to the unmodulated carrier condition when $I_2 = 0$. As k and δ_2 both have values $\ll 1$, $E_{2.0}$ may be found with great accuracy from

$$E_{2.0} \cong E_1 \cdot \sqrt{\frac{X_2}{X_1}} \cdot \frac{k}{j\delta_2} \tag{34}$$

showing that the secondary carrier voltage is 90 degrees dephased with respect to E_1, in accordance with the common intermediate-circuit theory. Substituting (33) in (31) we find

$$I_2 = \frac{(E_2 - E_{2.0})(k^2 + j\delta_2)}{X_2(\delta_2 + j(1 - k^2))} \tag{35}$$

which for values of k and $\delta_2 \ll 1.0$ simplifies to

$$I_2 = \frac{\delta_2}{X_2}(E_2 - E_{2.0}). \tag{36}$$

Now if we want E_2 to be amplitude modulated according to a sine function, E_2 must be of the form

$$E_2 = E_{2.0}(1 + m \cdot \sin x) \tag{36a}$$

which substituted in (36) gives

$$I_2 = \frac{\delta_2}{X_2} \cdot E_{2.0} \cdot m \cdot \sin x. \tag{37}$$

This means that in order to raise the voltage from the carrier value

$E_{2.0}$ to $2 \times E_{2.0}$, i.e., for 100 per cent up modulation, we must make the generator G_2 supply the current

$$I_{2.0} = \frac{\delta_2}{X_2} \cdot E_{2.0}. \tag{38}$$

In order to reduce the voltage from $E_{2.0}$ to zero, i.e., for 100 per cent down modulation, we have to supply the current

$$-I_{2.0} = -\frac{\delta_2}{X_2} \cdot E_{2.0} \tag{39}$$

which is in phase opposition to the current (38) but of the same magnitude.

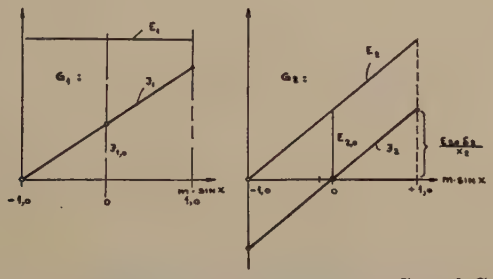

Fig. 7—Currents and voltages of the two generators G_1 and G_2 of Fig. 6 during a complete modulation cycle.

Substituting (36a) in (32) and reducing, using the same assumptions as above, we find for the primary current I_1

$$I_1 = I_{1.0}(1 + m \sin x) \tag{40}$$

where

$$I_{1.0} = E_1 \cdot \frac{k^2}{\delta_2 X_1} \tag{41}$$

is the carrier current supplied by generator G_1, and being in phase with the voltage E_1.

Thus the primary current is modulated in exactly the same way as the load current

The currents and voltages of the two generators are shown schematically in Fig. 7 as functions of $m \cdot \sin x$. For up modulation I_2 is in phase with E_2 and increases the current I_{L_2} in the secondary coil with the result that the back-induced voltage in circuit 1 is increased and the load impedance of generator G_1 is lowered. Just the opposite is hap-

pening when we reverse the phase of I_2 with respect to E_2, the current $-I_{2.0}$ being exactly sufficient to cancel the voltage induced in circuit 2 from circuit 1.

From (38) we find by substituting $E_{2.0}$

$$I_{2.0} = I_{1.0} \cdot \sqrt{\frac{X_1}{X_2}} \cdot \frac{\delta_2}{jk} \cdot \tag{42}$$

From (36a) and (42) it follows that

$$E_1 I_{1.0} = \left| E_{2.0} \cdot I_{2.0} \right| = W_c \tag{43}$$

which means that both generators have to be rated for the same power.

This is more clearly brought into light by considering in detail the energy supplied by the two generators.

According to (40) G_1 is supplying

$$\begin{aligned} W_1 &= E_1 \cdot I_{1.0} \cdot (1 + m \cdot \sin x) \\ &= W_c \cdot (1 + m \cdot \sin x) \end{aligned} \tag{44}$$

where W_c = carrier power. For full moduation ($m = 1.0$) the maximum power delivered by G_1 amounts to

$$W_{1\max} = 2 \cdot W_c.$$

The mean value of W_1 over a complete modulation cycle is

$$W_{1m} = W_c.$$

For generator G_2, the supplied power equals to

$$\begin{aligned} W_2 &= \left| E_{2.0} \cdot I_{2.0} \right| \cdot (1 + m \cdot \sin x) \cdot m \cdot \sin x \\ &= W_c (1 + m \cdot \sin x) \cdot m \cdot \sin x. \end{aligned} \tag{45}$$

The maximum value for full modulation is drawn simultaneously for generator G_2 and equals

$$W_{2\max} = 2W_c. \tag{46}$$

The mean value of W_2 equals

$$W_{2m} = W_c \cdot \frac{m^2}{2} \cdot \tag{47}$$

Thus the following may be concluded for this modulation scheme:

(1) The peak power $4 \times W_c$ is *automically* equally divided between the two generators, each generator delivering $2 \times W_c$ at peak modulation. This is true for any relative rating of the working voltages of the generators.

(2) The mean energy, taken over a complete modulation cycle, delivered by both generators equals (as it must)

$$W_c\left(1+\frac{m^2}{2}\right)$$

of which generator G_1 delivers the first constant term, and generator G_2 the second variable term due to the rise in the mean load power. Generator G_1 is thus truly a carrier generator and G_2 furnishes the side-band energy.

In Fig. 8 W_1 and W_2 for $m=1.0$ are plotted as functions of x. As is seen, in the negative half cycle of modulation (down) W_2 is zero, for

(1) $x=\pi$ because $I_2=0$

(2) $x=3\pi/2$ because $E_2=0$

(3) $x=2\pi$ because again $I_2=0$

W_2, it should be noted, must be negative for all values of m between $x=\pi$ and 2π, i.e., over the whole negative half cycle.

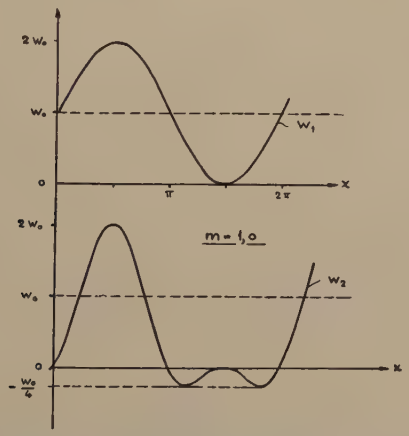

Fig. 8—Variations of carrier and side-band energy over a complete modulation cycle, and for full modulation ($m=1.0$).

From Fig. 8 it is seen how relatively small this power (absorbed by G_2) is in comparison with the positive peak power.

Concluding, we consider briefly the maxima and minima of W_1 and W_2, respectively:

W_1 for $x=\pi/2$ $W_{1\max}=W_c(m+1)$

for $x=3\pi/2$ $W_{1\min}=W_c(1-m)$

W_2 for $x=\pi/2$ $W_{2\max}=W_c(m^2+m)$

for $x=3\pi/2$ $W_{2\min}=W_c(m^2-m)$.

Further W_2 has two other minima if $m \geqq 0.5$ which are independent of m

$$\text{for} \quad m \cdot \sin x = -1/2, \qquad W_{2\min} = -\frac{W_c}{4}.$$

If $m < 1/2$ those last minima vanish. Thus the maximum power absorbed by G_2 during down modulation never exceeds $1/4 \cdot W_c$.

V. Mean Input Power Using Vacuum-Tube Generators

Making certain assumptions, we are able to calculate the mean input power as a function of m when G_1 and G_2 are vacuum-tube generators.

The following assumptions are made:

(1) Tube 1 has a constant grid-excitation voltage, so adjusted that the maximum power 2 W_c is delivered with a high anode-voltage swing and corresponding high anode efficiency

(2) Tube 2 is so adjusted that the carrier-anode voltage $E_{2.0}$ gives no anode current. This of course is necessary in order to avoid carrier tube losses in tube 2.

(3) By grid excitation tube 2 gives an anode alternating current proportional to the grid alternating voltage and in phase with this voltage.

(4) For both tubes the direct anode currents are assumed to be proportional to the anode alternating-current values irrespective of phase.

Following these assumptions, the input direct current to tube 1 is independent of modulation; the same is then true for the anode input power if the anode direct voltage is constant. The input power for tube 1 is thus

$$W_{1i} = \frac{W_c}{\eta_\sigma} \tag{48}$$

where W_c = carrier power and η_σ = anode efficiency of tube 1.

For the tube generator No. 2 the anode direct current according to the assumptions rises from zero to the maximum value both for the up and the down modulation. Thus the ratio between maximum and mean input power for this tube equals $(2/\pi)\ m$. The maximum output power of tube 2 being $2 \times W_c$ and assuming the same maximum efficiency η_σ as for tube 1, the mean input power becomes

$$W_{2i} = \frac{2W_c}{\eta_\sigma} \cdot \frac{2}{\pi} \cdot m. \tag{49}$$

The total mean input power to both generators thus amounts to

$$W_i = \frac{W_c}{\eta_\sigma}\left(1 + \frac{4}{\pi}m\right). \tag{50}$$

Tube 1 may in this scheme be class C operated as the grid voltage is constant. A higher net anode efficiency of 70 per cent can then be realized. For a 100-kilowatt carrier energy we then find

$$W_i = 143 + 181.5 \cdot m \text{ kw.}$$

for $m = 30\%$ $W_i = 197.5$ kw.

for $m = 100\%$ $W_i = 324.5$ kw.

The result of these purely theoretical calculations is shown as a dotted line in the earlier Fig. 2 in this paper. For $m = 30$ per cent the input power lies between those for systems (c), (d), and (e) earlier described.

The peak power, however, is higher for this system, as follows:

7 % higher than for system (b)
13 % higher than for system (c)
10 % higher than for system (d)
27.5% higher than for system (e_2)

VI. Requirements to Tube Characteristics

In Section IV we have shown that, if the side-band generator is supplying a current I_2 in the way described, and if the carrier generator keeps a constant terminal voltage E_1, then pure amplitude modulation is obtained as shown.

The next step is to consider if, using normal transmitting tubes, these conditions can be realized for the supplying of power.

Generator G_1 has to work with a constant anode alternating voltage independent of the load in the anode circuit. As is well known, this can only be realized approximately as the voltage necessarily must increase when the load impedance is increased, due to the internal tube resistance. Thus we get a sloping line for E_1, instead of the horizontal shown in Fig. 7. To counteract this rising voltage, one obvious remedy is well known, i.e., to bias the grid circuit with a voltage drop from the grid current of the tube. It is also possible to use a separate rectifier which is coupled to the anode circuit, and so adjusted that its direct-current output shows the desired dependence on the anode voltage.

Generator G_2 is subject to two separate restrictions.

(1) It must draw zero anode current when in carrier condition. In this condition the anode alternating voltage has a definite value $E_{2.0}$

and the grid alternating voltage is zero. Calling V_{a2} the anode direct voltage of tube 2, the required condition is obtained if the grid-bias voltage is chosen

$$\geqq \frac{1}{\mu}\left\{V_{a2} + \sqrt{2}\cdot E_{2.0}\right\}$$

where μ is the amplification factor of the tube.

(2) The anode alternating current (root-mean-square) must be proportional to the grid alternating voltage, also when this voltage is turned 180 degrees in phase. When the tube is biased according to (1) this condition is not to be realized without certain precautions, if we use triodes. However, if we use tetrodes, or pentodes, the requirement is a priori fulfilled, because then the anode current does not depend on the anode voltage, or at least only to a very small extent.

This difficulty is clearly brought to light if we consider for a moment some of the equations from the classical tube-oscillator theory. We use the following notations:

V_a = anode direct voltage

V_g = grid direct voltage

E_a = amplitude of anode alternating voltage

E_g = amplitude of grid alternating voltage.

Momentary voltages at grid and anode are then

$$\left.\begin{aligned} v_g &= -V_g + E_g \cos x \\ v_a &= V_a + E_a \cos x \end{aligned}\right\} \tag{51}$$

where E_g and E_a are taken to be in phase. The anode current is then some function of the controlling voltage

$$v_c = v_g + \frac{v_a}{\mu} = \left(E_g + \frac{E_a}{\mu}\right)(\cos x - \cos\theta) \tag{52}$$

where θ is half the pulse angle given by

$$\cos\theta = \frac{V_g - V_a/\mu}{E_g + E_a/\mu}\,. \tag{53}$$

Now anode current is flowing only if $v_c > 0$ which means that $\cos\theta$ must lie between -1 and $+1$ ($\theta = 0 - \pi$). As we have seen the condition (1) above requires that

$$V_g = \frac{V_a + E_a}{\mu} \cdot \tag{54}$$

Hence,

$$\cos\theta = \frac{E_a/\mu}{E_g + E_a/\mu} \cdot \tag{55}$$

From this we see that, if the tube is biased according to (54), anode current will flow if we increase E_g from zero to positive values, the angle θ increasing from 0 to $\pi/2$ and the last value being reached for $E_g = \infty$.

Anode and grid voltages are then in phase, and the tube operates for down modulation.

If we now turn the phase of E_g 180 degrees with respect to E_a, in order to obtain up modulation, no anode current will flow in the interval of E_g from 0 to $-2E_a/\mu$, because then $|\cos\theta| > 1$.

From values of $E_g = -2E_a/\mu$ and further, anode currents begin to flow again, as is easily seen. Such an operation would of course, be useless, due to prohibitive distortion.

Assuming the simplest case, that the anode current is directly proportional to the controlling voltage V_c, we find for the anode direct current I_a and the amplitude of the fundamental, $I_{a,1}$, respectively,

$$I_a = \text{proportional to } \left(E_g + \frac{E_a}{\mu}\right)\frac{1}{\pi}(\sin\theta - \theta\cos\theta) \tag{56}$$

$$I_{a1} = \text{proportional to } \left(E_g + \frac{E_a}{\mu}\right)\frac{1}{\pi}(\theta - \sin\theta\cos\theta) \tag{57}$$

which show that there is no simple exact linear relationship between currents and E_g to be expected.

If we use a tetrode or a pentode, however, (52) and (53) become, respectively,

$$\left.\begin{aligned} v_c = v_g + \frac{V_s}{\mu} &= E_g(\cos x - \cos\theta) \\ \cos\theta &= \frac{V_g - \dfrac{V_s}{\mu}}{E_g} \end{aligned}\right\} \tag{58}$$

where V_s is the constant screen-grid voltage and μ the screen amplification factor. In the same way (56) and (57) simplify to

$$\left.\begin{aligned} I_a &= \text{proportional to } E_g/\pi(\sin\theta - \theta\cos\theta) \\ I_{a1} &= \text{proportional to } E_g/\pi(\theta - \sin\theta\cos\theta) \end{aligned}\right\}. \tag{59}$$

Now we can easily satisfy the requirement that I_{a1} be proportional to E_g, by making θ independent of the grid voltage E_g. This requires

$$V_g = \frac{V_s}{\mu}, \qquad \text{i.e.,} \qquad \cos\theta = 0$$

or in other words, the well-known class B operation.

We can obtain the same result also with common triodes if we superpose in the grid the voltage

$$-\frac{E_a}{\mu}\cos x. \tag{60}$$

Adding this to the first equation of (51) we find

$$\left.\begin{aligned} v_c &= E_g(\cos x - \cos\theta) \\ \cos\theta &= \frac{V_g - \dfrac{V_a}{\mu}}{E_g} \end{aligned}\right\} \tag{61}$$

which are just the same equations as for the tetrode case, with V_a substituted for V_s.

If we now want an exact linear relation between E_g and I_{a1} we choose

$$V_g = \frac{V_a}{\mu}$$

and operate the tube as a class B amplifier. Thus we see that if we cancel the anode retroaction voltage, the triode also gives an anode current which is independent of the anode alternating voltage.

The necessary adjustment for this operation is simple. With the tube unexcited we adjust for correct anode and grid direct voltages and note the anode rest direct current. Anode alternating voltage is then adjusted according to the carrier condition, and the anode direct current will show a certain rise. Then the compensating voltage is introduced in the grid circuit and adjusted so that the anode direct current is reduced to the original rest current noted.

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NOTES ON THE IMPEDANCE OF A CARBON MICROPHONE*

By

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WHETHER or not the impedance of a carbon microphone is identical with its direct-current resistance has long been a question among radio engineers. We shall attempt a solution of this problem through the equivalent circuit of the microphone, transformer, and load. The treatment given will be in terms of a single-button microphone, but the results will obviously be just as applicable to the double-button type.

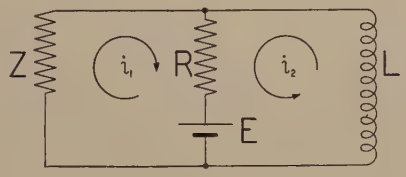

Fig. 1

The variation of resistance of the microphone with displacement, x, of the diaphragm from its equilibrium position is given by

$$R = R_0 + R_1 x + \cdots, \tag{1}$$

where R_0 is the direct-current resistance of the button, and powers of x beyond the first have been dropped. If a sound wave of angular frequency ω strikes the diaphragm, the displacement will be $x = x_0 \cos \omega t$ which with (1) gives

$$R = R_0 + R_1 x_0 \cos \omega t = R_0(1 + a \cos \omega t) \tag{2}$$

where $a = R_1 x_0 / R_0$ and $a \ll 1$.

Fig. 1 gives the equivalent circuit of the microphone coupled to a load of reflected impedance Z, by a perfect transformer. L is assumed of infinite inductance and zero resistance. We shall solve the mesh equations for this circuit. In the first branch,

$$I_1 Z + (I_1 + I_2)R = E. \tag{3}$$

In the second branch, I_2 will be the average value of E/R, because of the infinite inductance of the branch. From (2),

* Decimal classification: R385.5. Original manuscript received by the Institute, February 6, 1936.

$$I_2 = \overline{\frac{E}{R_0(1 + a\cos\omega t)}} = \frac{E}{R_0}\overline{(1 - a\cos\omega t + \cdots)} = \frac{E}{R_0} \qquad (4)$$

the bar over a quantity indicating its average value. Solving (3) and (4) for I_1 gives, with (2), since $a \ll 1$,

$$I_1 = -\frac{a\cos\omega t}{Z + R_0}E.$$

The impedance of a source may be considered as the value of the impedance into which it delivers maximum power. As the source is nonreactive, it is apparent that Z must be a pure resistance. Then the power dissipated in the load is

$$P = I_1^2 Z = a^2E^2\cos^2\omega t\frac{Z}{(Z + R_0)^2}.$$

Maximizing P with respect to Z, we set dP/dZ equal to zero:

$$dP/dZ = a^2E^2\cos^2\omega t\frac{(Z + R_0)^2 - 2Z(Z + R_0)}{(Z + R_0)^4} = 0$$

which gives,

$$Z = R_0.$$

In other words, the impedance of a carbon microphone is identical with its direct-current resistance.

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THE CAUSES FOR THE INCREASE OF THE ADMITTANCES OF MODERN HIGH-FREQUENCY AMPLIFIER TUBES ON SHORT WAVES*

By

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Summary—*By recent measurements of input loss, output loss, and feed-back capacitance of modern high-frequency amplifier tubes (pentodes) up to 300 megacycles, a considerable increase of these values in the short-wave range has become manifest. Contrary to the opinion, expressed in several recent publications, for many types of valves the main cause of this increase must not be sought in electron-transit-time effects, but in the action of capacitances, mutual inductances, and self-inductances of the tube electrodes and of their leads within and without the tubes. A general theory of the effect of these quantities on input admittance, output admittance, feed-back admittance, and mutual admittance is put forward for tetrodes, pentodes, hexodes, etc., used as high-frequency amplifiers. By three series of measurements it is shown, that about one to two thirds of the input damping of modern European high-frequency valves of normal dimensions on short waves must be ascribed to inductive effects and not to transit times. Several measurements on transit-time effects are described, showing that the transit time between the input grid and the screen grid may not be neglected as compared with the transit time between the cathode and the input grid. Theoretical formulas for inductive effects are well checked and those for transit-time effects are not so well checked by measurements. Causes for the latter deviations are given. Output admittance and feed-back admittance are almost wholly due to inductive effects in the short-wave region, as shown by several measurements described.*

I. Introduction

RECENT measurements[1,2,3] have shown a marked increase of input loss, output loss, and feed-back capacitance of modern high-frequency amplifier valves of normal dimensions. Valves AF3 and AF7 of Philips, Telefunken, Tungsram, and Valvo, are representative since they are high-frequency pentodes of the variable- and fixed-bias type. Some values of measured input admittances are given here (mean values taken from several valves).

In Table I, R_e (cold) is the input parallel resistance while the cathode is cold, R_e (biased) is the value with heater and normal positive voltages and with such grid bias that no anode current flows,

* Decimal classification: R262. Original manuscript received by the Institute, July 14, 1937; revised manuscript received by the Institute, November 20, 1937.

[1] *Elek. Nach. Tech.*, vol. 12, pp. 347–354; November, (1935).
[2] *Elek. Nach. Tech.*, vol. 13, pp. 260–268; August, (1936).
[3] *Elek. Nach. Tech.*, vol. 14, pp. 75–80; March, (1936), (with extensive bibliography).

R_e (normal) is the value at normal operating conditions (bias -3 volts and mutual conductance 1.8 milliamperes per volt with the AF3 whereas these values are -2 and 2.0 milliamperes per volt for the AF7), R_e (active) is given by $1/R_e$ (active) $= 1/R_e$ (normal) $- 1/R_e$ (biased). C_e(cold) and C_e (normal) are the values of input capacitance

TABLE I

Valve type	Wavelength m	R_e (cold) Megohms	R_e (biased) Megohms	R_e (normal) Megohms	R_e (active) Megohms	C_e(normal) $\mu\mu f$	C_e (cold) $\mu\mu f$
AF3	230	3.9	4.7	3.3	12	7.0	6.1
AF3	39.5	1.6	2.6	0.38	0.45	7.0	6.1
AF3	21.2	0.74	1.5	0.108	0.116	7.0	6.1
AF3	12.4	0.40	0.54	0.036	0.039	7.0	6.1
AF3	5.6	0.19	0.21	0.0086	0.0090	7.0	6.1
AF7	230	2.9	6.0	2.1	3.3	8.1	6.6
AF7	39.5	1.5	2.7	0.26	0.29	8.1	6.6
AF7	21.2	0.69	1.4	0.070	0.074	8.1	6.6
AF7	12.4	0.35	0.84	0.023	0.024	8.1	6.6
AF7	5.6	0.11	0.12	0.0045	0.0046	8.1	6.6

under cold-cathode and under normal operating conditions, respectively (mean values, taken from several valves).

Similar measurements for the output parallel resistance are given in Table II. The value of C_a (cold) is practically equal to C_a (normal).

TABLE II

Valve	Wavelength meters	R_a (cold) megohms	R_a (biased) megohms	R_a (normal) megohms	R_a (active) megohms
AF3	62.5	0.75	0.67	0.43	1.2
AF3	20.4	0.35	0.29	0.19	0.54
AF3	5.05	0.041	0.038	0.022	0.054
AF7	62.5	1.12	1.12	0.68	1.7
AF7	20.4	0.49	0.45	0.25	0.58
AF7	5.05	0.046	0.041	0.024	0.058

From these tables it is clear, that input parallel resistance R_e (normal) is much more unfavorable on short waves (below say 25 meters) than output parallel resistance R_a (normal). Feed-back admittance is inductive on short waves and may be compensated by a suitable capacitance from grid to anode. It is given by the empirical expression

$$Y_{ag}' = j\omega(C_{ag} - A\omega^2)$$

where C_{ag} is about 0.003 micromicrofarad for the valve AF3 and $A = 7 \cdot 10^{-19}$ (Dimension: $\mu\mu f/(c/s)^2$). Means for reducing A on short waves by suitable mutual inductances between electrode leads were given previously.[4]

It was the generally valid opinion, that most of the input damping must be ascribed to electron-transit-time effects on short waves. One of the prevalent aims of this article is to show that this opinion must

[4] *Op. cit.*, p. 267.

be considered erroneous in so far as it concerns modern European valves of normal dimensions. A very important contribution to the input damping comes from inductive effects, associated with the lengths of the electrode leads within the valves. The rôle of transit-time effects is not predominant.

Previously[1] it was shown that anode admittance and feed-back admittance must be wholly ascribed to inductive effects in the short-wave region. Van der Pol and Posthumus of this laboratory first drew attention to the rôle of impedances in the valve leads, and studied them theoretically and experimentally in 1933.

II. General Theory of the Tetrode as a High-Frequency Amplifier

Fig. 1 shows the tetrode scheme. The meaning of some of the symbols is as follows: i indicates alternating currents (effective value),

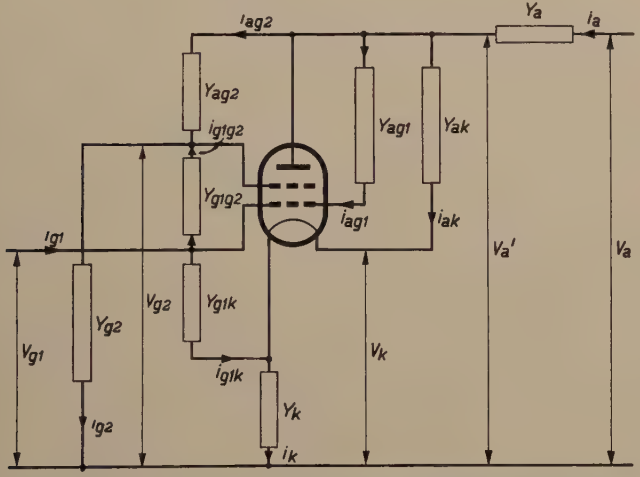

Fig. 1—Schematic of tetrode as a high-frequency amplifier. i = currents, V = voltages (both effective values), and Y = admittances. Arrows indicate alternating phase relations of the currents.

V = alternating volts against the earth (chassis). As to the indexes, occurring with these symbols, $g1$ is the input grid, $g2$ the screen grid, k the cathode, and a the anode. The symbol Y is an admittance, such as Y_{ak}, the admittance between anode and cathode, Y_k the admittance between the cathode and the earth, Y_{g2} the admittance between the screen grid and the earth (chassis). M (not shown in Fig. 1) indicates a coefficient of mutual inductance, such as M_{g2k}, the mutual inductance between the leads to the cathode and the screen grid. The

mutual inductance between the grid lead and the other leads is neglected, the grid electrode being on top of the bulb. (Some valves on the British market form an exception.) The influence of this inductance can easily be incorporated in our theory. The symbol ω is the angular frequency and $j=+\sqrt{-1}$. S is the mutual conductance, such as S_{g2} the mutual conductance between the input grid and the screen grid, and S_a between the input grid and the anode. The symbol V_a' is the anode voltage in the tube, measured on the anode itself, against the earth, whereas V_a is the voltage between the tube electrode pin and the earth. This differentiation between the voltage on the electrode valve pin and on the valve electrode itself could of course also be made with respect to the other valve electrodes. Numerically, however, it is of little consequence with these electrodes besides the anode.[5]

One finds

$$\left.\begin{aligned} V_a' &= V_a - i_a/Y_a + j\omega M_{ag2} i_{g2} + j\omega M_{ak} i_k; \\ V_{g2} &= i_{g2}/Y_{g2} + j\omega M_{g2k} i_k - j\omega M_{ag2} i_a; \\ V_k &= i_k/Y_k + j\omega M_{g2k} i_{g2} - j\omega M_{ak} i_a; \end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned} i_{g1g2} &= (V_{g1} - V_{g2}) Y_{g2g1}; \\ i_{g1k} &= (V_{g1} - V_k) Y_{g1k}; \\ i_{ag1} &= (V_a' - V_{g1}) Y_{ag1}; \\ i_{ag2} &= (V_a' - V_{g2}) Y_{ag2}; \\ i_{ak} &= (V_a' - V_k) Y_{ak}; \end{aligned}\right\} \quad (2)$$

$$\left.\begin{aligned} i_{g1} &= i_{g1g2} + i_{g1k} - i_{ag1}; \\ i_a &= S_a(V_{g1} - V_k) + i_{ag2} + i_{ak} + i_{ag1}; \\ i_{g2} &= i_{ag2} + i_{g1g2} - S_{g2}(V_{g1} - V_k); \\ i_k &= i_{g1k} + i_{ak} + (S_a + S_{g2})(V_{g1} - V_k). \end{aligned}\right\} \quad (3)$$

In these twelve equations there are fourteen variables: V_a' V_a, V_{g2}, V_k, V_{g1}, i_{g1g2}, i_{g1k}, i_{ag1}, i_{ag2}, i_{ak}, i_{g1}, i_a, i_{g2}, and i_k. We shall express i_{g1} and i_a by V_{g1} and V_a. The actual resolution of the 12 equations (1), (2), and (3) is very tiresome and will be omitted here for the sake of brevity. The resulting two equations are

$$i_{g1} = (Y_{g1g2} + Y_{g1k} + Y_{ag1} - P)V_{g1} - (Y_{ag1} + Q)V_a; \quad (4)$$

$$i_a = (S_a - Y_{ag1} - A)V_{g1} + (Y_{ag2} + Y_{ak} + Y_{ag1} - B)V_a. \quad (5)$$

In (4) and (5) the following symbols have been introduced:

[5] *Op. cit.*, p. 266.

$$P = \{Y_{g1g2}(\beta_2 a_1 - \beta_1 a_2) - Y_{g1k}(\alpha_2 a_1 - \alpha_1 a_2)\}(\alpha_1\beta_2 - \beta_1\alpha_2)^{-1};$$

$$A = \{Y_{ag2}(\beta_2 a_1 - \beta_1 a_2) - (Y_{ak} + S_a)(\alpha_2 a_1 - \alpha_1 a_2)\}(\alpha_1\beta_2 - \beta_1\alpha_2)^{-1};$$

$$Q = \{Y_{g1g2}(\beta_2 b_1 - \beta_1 b_2) - Y_{g1k}(\alpha_2 b_1 - \alpha_1 b_2)\}(\alpha_1\beta_2 - \beta_1\alpha_2)^{-1};$$

$$B = \{Y_{ag2}(\beta_2 b_1 - \beta_1 b_2) - (Y_{ak} + S_a)(\alpha_2 b_1 - \alpha_1 b_2)\}(\alpha_1\beta_2 - \beta_1\alpha_2)^{-1};$$

$$\alpha_1 = Y_{g2} + Y_{ag2} + Y_{g1g2} - j\omega M_{ag2}Y_{ag2}Y_{g2};$$

$$\alpha_2 = j\omega M_{g2k}Y_k(Y_{ag2} + Y_{g1g2}) - j\omega M_{ak}Y_kY_{ag2};$$

$$a_1 = Y_{g1g2} - S_{g2} + j\omega M_{g2k}Y_{g2}(Y_{g1k} + S_a + S_{g2}) - j\omega M_{ag2}Y_{g2}(S_a - Y_{ag1});$$

$$a_2 = Y_{g1k} + S_a + S_{g2} + j\omega M_{g2k}Y_k(Y_{g1g2} - S_{g2}) - j\omega M_{ak}Y_k(S_a - Y_{ag1});$$

$$\beta_1 = j\omega M_{g2k}Y_{g2}(Y_{g1k} + Y_{ak} + S_a + S_{g2}) - j\omega M_{ag2}Y_{g2}(Y_{ak} + S_a) - S_{g2};$$

$$\beta_2 = Y_k + Y_{g1k} + Y_{ak} + S_a + S_{g2} - j\omega M_{ak}(Y_{ak} + S_a)Y_k;$$

$$b_1 = Y_{ag2} + j\omega M_{g2k}Y_{g2}Y_{ak} - j\omega M_{ag2}Y_{g2}(Y_{ag2} + Y_{ak} + Y_{ag1});$$

$$b_2 = Y_{ak} + j\omega M_{g2k}Y_kY_{ag2} - j\omega M_{ak}Y_k(Y_{ag2} + Y_{ak} + Y_{ag1}).$$

Formulas (4) and (5) are not yet very suited to practical application. It is useful to develop these expressions in powers of ω. One then can immediately see which terms become most important if the wavelength is decreased. Calculations are again very tiresome and for the sake of briefness only the results are given here:

$$Y_k = 1/j\omega L_k;\quad Y_{g2} = 1/j\omega L_{g2};\quad Y_{ak} = j\omega C_{ak} + 1/R_{ak};\quad Y_{ag2} = j\omega C_{ag2};$$

$$Y_{g1k} = j\omega C_{g1k} + 1/R_{g1k};\quad Y_{g1g2} = j\omega C_{g1g2};\quad Y_{ag1} = j\omega C_{ag1}.$$

Here L is the coefficient of self-inductance of the lead inside the valve between the prong and the electrode. C is the capacitance between the electrodes, and R is a resistance.

We use the following abbreviations:

$$Y_{ag}' = Y_{ag1} + Q;\quad Y_i' = Y_i - P;$$

$$Y_0' = Y_0 - B;\quad S_a' = S_a - (A + Y_{ag1});$$

$$Y_i = Y_{g1k} + Y_{g1g2} + Y_{ag1};\quad Y_0 = Y_{ak} + Y_{ag2} + Y_{ag1}.$$

Then,

$$\left.\begin{aligned}
Y_{ag}' &= j\omega C_{ag1} - j\omega^3\{C_{ag2}(L_{g2}C_{g2g1} + M_{g2k}C_{kg1}) + C_{ak}(M_{g2k}C_{g2g1} \\
&\quad + L_kC_{kg1}) - (C_{ag2} + C_{ak})(M_{ag2}C_{g2g1} + M_{ak}C_{kg1})\} + \cdots; \\
Y_i' &= Y_i + \omega^2\{(S_a + S_{g2})(L_kC_{kg1} + M_{g2k}C_{g2g1}) - S_{g2}(M_{g2k}C_{kg1} \\
&\quad + L_{g2}C_{g2g1}) - S_a(M_{ak}C_{kg1} + M_{ag2}C_{g2g1})\} + \cdots; \\
Y_0' &= Y_0 + \omega^2 S_a\{C_{ak}L_k + C_{ag2}M_{g2k} - (C_{ak} + C_{ag2})M_{ak}\} + \cdots; \\
S_a' &= S_a - j\omega C_{ag1} + j\omega S_a\{-L_k(S_a + S_{g2}) \\
&\quad + M_{g2k}S_{g2} + M_{ak}S_a\} + \cdots.
\end{aligned}\right\} \quad (5a)$$

The proportionality of $1/R_e$ (active) and of $1/R_a$ (active), contained in the real parts of Y_i' and Y_0', to ω^2 may be understood on a general theoretical basis. Consider any agglomeration of inductances, resistances, and capacitances and take the admittance between any two terminals. This admittance Y may be resolved into a resistive part, parallel to a capacitive part: $Y=1/R+j\omega C$. In the complicated expression for Y the frequency ω enters always and only in the combination $j\omega$. No other possibility exists for the imaginary unit j to enter into the formulas. Hence, developing Y into powers of ω, the *real* quantities $1/R$ and C can only contain *even* powers of $j\omega$ and hence of ω. This is exactly what (5a) shows,

III. Extension of the above Theory to Pentodes and Hexodes Used as High-Frequency Amplifiers

If one considers attentively the arrangement of (5a) and takes into account the new electrodes and leads with pentodes and hexodes, which are additional to the ones considered above, it is not difficult to obtain expressions for the admittances of the latter valves without further calculations.

The tetrode considered above has no metal screen or metalization outside the bulb. In Europe, such a screen or metalization (m) is, usually present and has a separate lead to a connection on the tube socket. We have to take account of the capacitance of the metalization to the other tube electrodes and of the self-inductance and mutual inductance of the lead. Formulas (5a) are slightly extended for such a tetrode with metalization. A high-frequency pentode has, moreover, a suppressor-grid electrode and lead. This is indicated by $g3$. We get, instead of (5a), for a metalized high-frequency pentode

$$\left.\begin{aligned} Y_{ag}' = {} & j\omega C_{ag1} - j\omega^3\{C_{am}(L_mC_{mg1}+M_{mg3}C_{g3g1}+M_{mg2}C_{g2g1}+M_{mk}C_{kg1}) \\ & +C_{ag3}(M_{g3m}C_{mg1}+L_{g3}C_{g3g1}+M_{g3g2}C_{g2g1}+M_{g3k}C_{kg1}) \\ & +C_{ag2}(M_{g2m}C_{mg1}+M_{g2g3}C_{g3g1}+L_{g2}C_{g2g1}+M_{g2k}C_{kg1}) \\ & +C_{ak}(M_{km}C_{mg1}+M_{kg3}C_{g3g1}+M_{kg2}C_{g2g1}+L_kC_{kg1}) \\ & -(C_{am}+C_{ag3}+C_{ag2}+C_{ak})(M_{am}C_{mg1} \\ & +M_{ag3}C_{g3g1}+M_{ag2}C_{g2g1}+M_{ak}C_{kg1})\} + \cdots; \end{aligned}\right\} \quad (6)$$

$$\left.\begin{aligned} Y_i' = {} & Y_i+\omega^2\{(S_a+S_{g2})(L_kC_{kg1}+M_{kg2}C_{g2g1}+M_{kg3}C_{g3g1}+M_{km}C_{mg1}) \\ & -S_{g2}(M_{g2k}C_{kg1}+L_{g2}C_{g2g1}+M_{g2g3}C_{g3g1}+M_{g3m}C_{mg1}) \\ & -S_a(M_{ak}C_{kg1}+M_{ag2}C_{g2g1}+M_{ag3}C_{g3g1}+M_{am}C_{mg1})\} + \cdots; \end{aligned}\right\} \quad (7)$$

$$\left.\begin{aligned} Y_0' = {} & Y_0+\omega^2S_a\{C_{ak}L_k+C_{ag2}M_{g2k}+C_{ag3}M_{g3k}+C_{am}M_{mk} \\ & -(C_{ak}+C_{ag2}+C_{ag3}+C_{am})M_{ak}\} + \cdots. \end{aligned}\right\} \quad (8)$$

Equation (8) was published previously[1] when discussing experimental results on anode admittance.

The equation for the mutual conductance of a high-frequency pentode is exactly equal to the corresponding expression (5a).

With a high-frequency amplifier hexode, the suppressor grid is absent and a second negative grid 3 is introduced. Grid 4 is a screen. The mutual conductance of grid 3 to the anode is S_{g3a}. We obtain

$$\left.\begin{aligned} Y_{ag}' = {} & j\omega C_{ag1} - j\omega^3 \{ C_{am}(L_m C_{mg1} + M_{mg4} C_{g4g1} + M_{g3m} C_{g3g1} + M_{mg2} C_{g2g1} \\ & + M_{mk} C_{kg1}) + C_{ag4}(M_{mg4} C_{mg1} + L_{g4} C_{g4g1} + M_{g4g3} C_{g3g1} \\ & + M_{g4g2} C_{g2g1} + M_{g4k} C_{kg1}) + C_{ag3}(M_{g3m} C_{mg1} + M_{g3g4} C_{g4g1} \\ & + L_{g3} C_{g3g1} + M_{g3g2} C_{g2g1} + M_{g3k} C_{kg1}) + C_{ag2}(M_{g2m} C_{mg1} \\ & + M_{g2g4} C_{g4g1} + M_{g2g3} C_{g3g1} + L_{g2} C_{g2g1} + M_{g2k} C_{kg1}) \\ & + C_{ak}(M_{mk} C_{mg1} + M_{g4k} C_{g4g1} + M_{kg3} C_{g3g1} + M_{kg2} C_{g2g1} + L_k C_{kg1}) \\ & - (C_{am} + C_{ag4} + C_{ag3} + C_{ag2} + C_{ak})(M_{am} C_{mg1} + M_{ag4} C_{g4g1} \\ & + M_{ag3} C_{g3g1} + M_{ag2} C_{g2g1} + M_{ak} C_{kg1}) \} + \cdots ; \end{aligned}\right\} \quad (9)$$

$$\left.\begin{aligned} Y_i' = {} & Y_i + \omega^2 \{ (S_a + S_{g2} + S_{g4})(L_k C_{kg1} + M_{kg2} C_{g2g1} + M_{kg3} C_{g3g1} \\ & + M_{kg4} C_{g4g1} + M_{km} C_{mg1}) \\ & - S_{g2}(M_{g2k} C_{kg1} + L_{g2} C_{g2g1} + M_{g2g3} C_{g3g1} + M_{g2g4} C_{g4g1} + M_{g2m} C_{mg1}) \\ & - S_{g4}(M_{g4k} C_{kg1} + M_{g4g2} C_{g2g1} + M_{g4g3} C_{g3g1} + L_{g4} C_{g4g1} + M_{g4m} C_{mg1}) \\ & - S_a(M_{ak} C_{kg1} + M_{ag2} C_{g2g1} + M_{ag3} C_{g3g1} + C_{g4g1} M_{g4a} \\ & + C_{mg1} M_{am}) \} + \cdots ; \end{aligned}\right\} \quad (10)$$

$$\left.\begin{aligned} Y_0' = {} & Y_0 + \omega^2 (S_{g1a} + S_{g3a}) \{ C_{ak} L_k + C_{ag2} M_{g2k} + C_{ag3} M_{g3k} \\ & + C_{ag4} M_{g4k} + C_{am} M_{mk} - (C_{ak} + C_{ag2} + C_{ag3} + C_{ag4} + C_{am}) M_{ak} \} \\ & - \omega^2 S_{g3a} \{ C_{ak} M_{kg3} + C_{ag2} M_{g2g3} + C_{ag3} L_{g3} + C_{ag4} M_{g4g3} \\ & + C_{am} M_{mg3} - (C_{ak} + C_{ag2} + C_{ag3} + C_{ag4} + C_{am}) M_{ag3} \} + \cdots ; \end{aligned}\right\} \quad (11)$$

$$\left.\begin{aligned} S_a' = {} & S_a - j\omega C_{ag1} + j\omega \{ -L_k(S_a + S_{g2} + S_{g4}) + M_{g2k} S_{g2} \\ & + M_{g4k} S_{g4} + M_{ak} S_a \} S_a + \cdots , \end{aligned}\right\} \quad (12)$$

where S_a stands for S_{g1a}, S_{g2} for S_{g1g2}, and S_{g4} for S_{g1g4}.

On the basis of these expressions it is easy to obtain formulas for any type of valve, such as high-frequency heptodes; expressions were also derived for mixing valves. A discussion of these formulas falls out of the scope of this article and will be given later together with the results of measurements.

IV. Discussion of the Obtained Expressions

Some general remarks on the above expressions may be useful. All terms, proportional to the lowest powers of ω were written down in the above expressions, regardless of their relative magnitude. This magnitude depends on the respective values of the capacitances and inductance coefficients. If we imagine the tubes mounted in a receiver chassis, the capacitances must be reckoned between the tube grids, cathode, anode, and metalization themselves, but also between the leads within the tubes and from tube to chassis. Of course no exact values can be given to each of these contributions, as concentrated capacitances

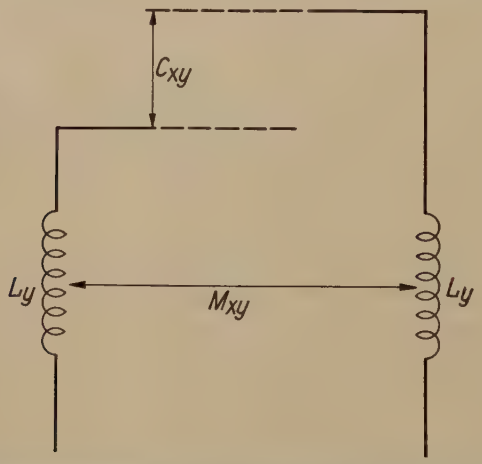

Fig. 2—Schematic indication of self-inductances, mutual inductance, and capacitance between the leads and electrodes marked x and y.

were assumed in the derivation of the formulas and induction coefficients between the leads (see Fig. 2). In reality, the capacitance between the leads shunts part of the inductances. As to the values of the inductances, to be inserted into the formulas, the same applies as for the capacitances, the leads must be taken up to the connection to the receiver chassis. In special cases, even parts of the chassis must be taken into account (see Section VII).

It is useful to restate here some well-known formulas for the self-inductances and mutual inductances of straight wires.

(a) Self-inductance of a wire, of length l and diameter d ($l \gg d$):

$$L = 2l\left(ln\frac{4l}{d} - 1\right)\cdot 10^{-9} \text{ henry.} \tag{13}$$

(b) Mutual inductance of two parallel wires, of length l and distance a ($l \gg a$):

$$M = 2l\left(ln\frac{2l}{a} - 1\right)\cdot 10^{-9} \text{ henry.} \tag{14}$$

(c) Mutual inductance of two straight wires, each of length l, having at one end the distance c and coinciding with their other ends ($l \gg c$):

$$M = 2l\left(ln\frac{2l}{c}\right)\cdot 10^{-9} \text{ henry.} \tag{15}$$

Applying (13), we have for a wire of 4 centimeters length and 0.05 centimeter diameter $L = 38\cdot 10^{-9}$ henry. In (14), if we take $l = 4$ centimenters and $a = 0.5$ centimeter, $M = 14\cdot 10^{-9}$ henry. If we take $l = 4$ centimeters and $c = 0.5$ centimeter in the case of (15) we get $M = 22\cdot 10^{-9}$ henry.

For the actual calculation of the several expressions set forth in Sections II and III, the capacitive values to be inserted must be known. We consider the valve AF3 as an example. $C_{ak}+C_{am}+C_{ag3}+C_{ag2} = C_0 = 7.6$ micromicrofarads; $C_{ak} = 0.1$ micromicrofarad; $C_{ag2} = 0.5$ micromicrofarad; $C_{ag3} = 3.5$ micromicrofarads; $C_{am} = 3.5$ micromicrofarads; $C_{g1g2}+C_{g1k}+C_{g1m} = 7.0$ micromicrofarads (under normal operation); $C_{g1g2} = 2.1$ micromicrofarads; $C_{g1k} = 3.4$ micromicrofarads; $C_{g1g3} = 0.5$ micromicrofarad; $C_{g1m} = 1$ micromicrofarad; $C_{ag1} = 0.002$ micromicrofarad. These values apply to an average valve of this type.

The mutual inductance between two leads within the tube depends very much on their distance; e.g., in the valve pinch or, with constructions without pinch, on their relative position. It is possible to choose this position, taking account of our formulas, so as to minimize certain inductive effects. This was successfully carried out in several instances (see Section VII). By inserting extra mutual inductances into the leads, some valve admittances could be favorably altered.

From the equations of Section III, together with (13), (14), and (15), it appears that in the short-wave region anode admittance and feed-back impedance are made worse if the leads of the suppressor grid and of the metalization of pentodes are connected to the cathode within the tube. Some of the relevant inductances are thereby increased from 1.5 to 3 times.

It should be stressed, that the mutual conductance does not enter into (5a), (6), and (9) for the feed-back admittance. Hence, values of the feed-back admittance taken with a cold cathode and under

normal operating conditions will differ only in as much as some capacitances, such as C_{kg1} and C_{g2g1} have different values under these circumstances. It will generally be sufficient to measure Y_{ag1}' with a cold cathode.

On very short waves a wavelength is finally attained, at which resonances may occur within the tubes. This may be shown from (5) together with the expressions for P, Q, A, and B. The denominator of these quantities consists of the expression $(\alpha_1\beta_2-\alpha_2\beta_1)$. If this denominator becomes zero, the tube admittances become infinite. Of course, our formulas do not hold in the immediate vicinity of this frequency. It is, however, possible to make a fairly satisfactory calculation of the frequencies, at which these resonances occur. As an example, take a tetrode without mutual inductance between the leads and with $C_{g2k}=0$. Then $\alpha_1\beta_2-\alpha_2\beta_1=(Y_{g2}+Y_{ag2}+Y_{g1g2})(Y_k+Y_{ak}+Y_{g1k}+S_a+S_{g2})$ and $Y_{g2}+Y_{ag2}+Y_{g1g2}=(j\omega L_{g2})^{-1}\left\{1-L_{g2}\omega^2(C_{ag2}+C_{g1g2})\right\}$. The resonance frequency follows approximately from $\omega^2L_{g2}(C_{ag2}+C_{g1g2})=1$. Taking $L_{g2}=4\cdot10^{-8}$ henry, $C_{ag2}=3$ micromicrofarads, $C_{g1g2}=2.3$ micromicrofarads, $\omega=2\cdot10^9$. The wavelength is about 1 meter. Experimentally we found, that tubes of normal dimensions showed resonances between 0.5 and 2 meters wavelength.

V. Measurements Leading to a Separation of the Causes of Input Admittance

According to (7) the input admittance Y_i' consists of a part Y_i and of a second part, proportional to ω^2. The former part Y_i contains the dielectric losses and the influence of electron-transit times, in the resistance R_{g1k}, whereas the latter part is exclusively due to inductive effects. Several measurements were taken, permitting a separation of the electronic part from the inductive part.

A tetrode, with measured input admittance on a wavelength of 6.3 meters, was measured again, inserting a small self-inductance of about 10^{-7} henry between the cathode prong and earth. The values of input parallel resistance R_e were as follows:

Anode current milliamperes	R_e without inserted extra self-inductive R_e	R_e with extra self-inductance
3	2450 ohms	1200 ohms
5	2050 ohms	990 ohms

By (5a) the additional input parallel resistance R_L due to a self-inductance in the cathode lead is given by

$$\frac{1}{R_L} = \omega^2(S_a + S_{g2})L_k C_{kg1}.$$

Taking $\omega = 3 \cdot 10^8$, $L_k = 10^{-7}$ henry, $S = S_a + S_{g2} = 6$ milliamperes per volt at 5 milliamperes anode current, and $C_{kg1} = 8$ micromicrofarads, the value of $R_L = 2300$ ohms. This is in sufficient concordance with the measured value of 1900 ohms, if one takes account of the possible errors in the determination of L_k and of C_{kg1}. Within the tube the cathode lead consists of 6 centimeters of wire of 0.5-millimeter diameter, corresponding to a self-inductance of $6 \cdot 10^{-8}$ henry. Calculating R_L due to this internal cathode lead, one finds, that more than one half of the measured input parallel resistance of 2450 ohms is due to the inductive effect of this lead. Hence, if we neglect the small compensating effects, due to screen-grid and anode-lead inductances, according to (5a), the electronic causes contribute less than one half to the total loss.

As a second experiment, tetrodes were taken, in some of which an additional capacitance of known value was inserted between the cathode and the input grid. Small condensers of the same type now used in octodes of type EK2 for compensating electron coupling were connected directly between the cathode and the grid inside the tube. We measured 4 valves with and 4 valves without extra capacitance. Mean values of these measurements were taken, in order to exclude uncontrolled tolerances of individual valves. The values of active input parallel resistance on a wavelength of 6.5 meters were:

TABLE III

Type	R_e (active) at 2 milliamperes anode current	R_e (active) at 4 milliamperes anode current
A without extra capacitance	3150 ohms	2170 ohms
B with extra capacitance	2280 ohms	1530 ohms

The values of C_{kg1} were determined for both types under operating conditions. With type *A* tubes these values were: at 2 milliamperes anode current $C_{kg1} = 7.0$ micromicrofarads and with 4 milliamperes anode current 7.5 micromicrofarads. The extra capacitance of type *B* was 4.6 micromicrofarads. Thus, at 4 milliamperes 4.6 micromicrofarads extra capacitance cause a decrease of input parallel resistance from 2170 to 1530 ohms and are equivalent to $(2170 \cdot 1530)/640 = 5200$ ohms parallel resistance. Hence, 7.5 micromicrofarads capacitance are equivalent to $4.6 \cdot 5200/7.5 = 3200$ ohms parallel resistance. The input parallel resistance due to electronic causes is $2170 \cdot 3200/1030 = 6700$

ohms. A similar calculation may be applied to the data at 2 milliamperes anode current. The final results are:

at 4 milliamperes: total input parallel resistance 2170 ohms
- 3200 ohms due to inductive effects
- 6700 ohms due to electron-transit times
- 5500 ohms due to inductance

at 2 milliamperes: total input parallel resistance 3150 ohms
- 7500 ohms due to electron-transit times.

The most important contribution to input-parallel-resistance loss is due to inductance and not electronic-transit effects, exactly as in the experiment first quoted again, neglecting the small compensating terms, due to screen-grid and anode-lead inductances (see (5a)).

From the above data the self-inductance of the cathode lead, with

$$S_a + S_{g2} = 6 \text{ milliamperes per volt amounts to}$$

$$L_k = \frac{3200}{\omega^2 S C_{kg1}} = 8 \cdot 10^{-8} \text{ henry.}$$

Taking a lead length of 6 centimeters and a diameter of 0.05 centitimeter, $L_k = 6.3 \cdot 10^{-8}$ henry, which represents a satisfactory check.

From (5a) and (7) it appears that the input loss, because of inductive effects, is made up of two parts, one positive, which contains the effect of the cathode lead, and a second part, containing the effects of the screen-grid and anode-lead inductances, which has a negative sign and hence may act as a compensation of the first part. Taking the case of a tetrode, dealt with by the second part of (5a), the positive real part of input admittance due to lead effects is $\omega^2(S_a+S_{g2})\ (L_kC_{kg1} + M_{g2k}C_{g2g1})$ and the negative part is $\omega^2\ [S_{g2}(M_{g2k}C_{g1g2}+L_{g2}C_{g2g1}) + S_a(M_{ak}C_{kg1}+M_{ag2}C_{g2g1})]$. From the discussion in Section IV it appears that mutual inductances between parallel wires, at a distance equal to ten times the diameter, are about one third of the self-inductances. Hence we might, as a first step towards simplification of the analysis, neglect the mutual inductance effects and we then obtain a positive real input admittance due to the leads of $\omega^2(S_a+S_{g2})L_kC_{kg1}$ and a negative real input admittance of $\omega^2 S_{g2}L_{g2}C_{g1g2}$. Taking the valve type

AF3 as a normal European high-frequency pentode, $S_a = 1.8$ milliampere per volt, $S_{g2} = 0.4$ milliampere per volt, $L_k = 4 \cdot 10^{-8}$ henry, $L_{g2} = 4 \cdot 10^{-8}$ henry, $C_{kg1} = 3.4$ micromicrofarads, and $C_{g1g2} = 2.1$ micromicrofarads. Hence the positive part is $\omega^2 30 \cdot 10^{-23}$ mhos and the negative part is $\omega^2 3.4 \cdot 10^{-23}$ mhos, which is only about one tenth of the positive part. This is the reason, which for simplicity, made us neglect the compensating effect of the screen-grid lead and anode-lead inductances on input admittance in the conclusions, drawn from the foregoing two experiments.

Fig. 3—Röntgen picture of two metalized valves, containing one (left) and two parallel systems, equal to those, used in the Philips EF5 type. Total length of valves on the picture is 95 millimeters.

In order to check this reasoning experimentally, we took a Philips high-frequency pentode valve (glass type FE5) with rather short leads. The data of this valve are: mutual conductance at anode current of 8 milliamperes (screen-grid current of 2.6 milliamperes) 1.7 milliamperes per volt, cold input capacitance 5.4 micromicrofarads, variable-μ type. (For a comparison, take the 58 tube, with a mutual conductance of 1.6 milliamperes per volt at 8.2 milliamperes anode current, 2.0 milliamperes screen current, and a cold input capacitance of 4.7 micromicrofarads.) The total outer dimensions of the EF5 are about equal to those of the 58 (see Fig. 3). Our tube factory made two valve types for us (see Fig. 3), one containing one system of the normal EF5 valve, with internal leads, bent as seen on the left of Fig. 3 and the other containing two normal EF5 systems in parallel, with the leads bent as seen on the Röntgen photograph at the right of Fig. 3. The

input grids are brought out on top of the valves. Base connections are equal to those of the normal EF5 type. The effects of the leads in the double valve are four times as large as those in the single valve, as the capacitances and slopes of the double valve are twice those of the single valve, whereas the lead inductances are approximately equal. If R_{el} is the input parallel resistance due to the electron-transit times, we have for the single valve a composite input parallel resistance $R'=R_{el}R_l/(R_{el}+R_l)$, where R_l is the input parallel resistance due to the lead effects. For the double valve we obtain $R''=\frac{1}{2}R_{el}\frac{1}{4}R_l/(\frac{1}{2}R_{el}+\frac{1}{4}R_l)$ as the transit-time effect for the double valve is half that for the single valve due to doubling of the mutual conductance (see (16)). Hence

$$R'/R''=2(1+R'/R_l)=2+2R_{el}/(R_{el}+R_l).$$

The measured active (see the legend to Table I) resistances were for two individual valves of each type:

single valve No. 1 R'	12100 ohms	double valve No. 1 R''	4450 ohms
single valve No. 2 R'	12900 ohms	double valve No. 2 R''	4650 ohms
single valve mean value	12500 ohms	double valve mean value	4550 ohms

The measurements were taken at 6.05 meters wavelength at 8 milliamperes anode current for the single valve and at 16 milliamperes anode current for the double valve. We find $R'/R''=2.75$ and $R'/R_l=3/8$, or $R_l=34{,}000$ ohms and $R_{el}=20{,}000$ ohms. The conclusion is, that in this valve, resembling the 58, a very considerable part of input loss on short waves is due to lead effects. In these experiments, the compensating effects of screen grid and anode leads are fully included, as all inductive lead effects are four times as large in the double valve compared with the single one. In the valve pinch the leads are respectively connected to the anode, suppressor grid, screen grid, two heating wires, cathode, suppressor grid, and anode. The second and the last lead end in the pinch. From these lead connections it is seen that the mutual inductance between the anode lead and the screen-grid lead is large and the mutual inductance between the screen lead and the cathode lead is small, both of which promotes the compensation of input loss due to the self-inductance of the cathode lead (see (5a)). But even in the case of this favorably compensated valve about 40 per cent of the total input loss is due to the leads.

It may be remarked that most of the measurements published by Ferris[7] and North[20] are carried out with triodes. In a triode the com-

[7] Refer to Bibliography.
[20] Refer to Bibliography.

pensation of the cathode-lead effect by the anode-lead inductance is much more effective than with tetrodes or pentodes of the types considered here. Hence in their cases the major part of total input loss might be due to electron-transit times and thus their experiments are favorable for studying transit-time effects. Our experiments, on the contrary, are specially suited, to show the inductive effects on input parallel resistance.

The above data (second experiment) lead to the conclusion, that with decreasing anode current the influence of self-inductance on input loss decreases faster than the influence of transit-time effects. This may be explained by theory, if we take account of the formula for the input parallel resistance R_{el} due to electrons[2]

$$\frac{1}{R_{el}} = \frac{1}{20} S(\omega\tau)^2. \tag{16}$$

In this equation S is the total mutual conductance of the input grid to the positive electrodes in amperes per volts and τ the transit time of electrons for the cathode-grid path in seconds. If we reduce S by negative grid bias, τ is increased. On the contrary, in $1/R_L = \omega^2 L_k S C_{kg1}$ the conductance S and the capacititance C_{kg1} are both decreased. Hence, $1/R_L$ decreases faster than $1/R_{el}$.

By the above results we are able to prove that it is useless, in valves of *normal* dimensions, such as the EF5 or the AF3, to decrease the cathode-grid clearance too much. In (16) S increases with decreasing distance, but τ decreases. In $1/R_L = \omega^2 L_k S C_{kg1}$ the conductance S and the capacitance C_{kg1} are both increased. Hence, $1/R_L$ increases much more than $1/R_{el}$ and the reduction of electronic loss by decreasing the the cathode-grid distance is overcompensated by the increase of inductive loss. Of course, if all valve dimensions are decreased (acorn valves), grid impedance may be increased.

For a physical interpretation of the inductive effects it may be interesting to consider the question: Where does the energy, due to increased input loss by self-inductance land? A simple calculation, which is here omitted for the sake of briefness, shows, that this loss energy is dissipated in the anode of the valve.

VI. Measurements on the Electronic Part of Input Admittance

The application of (16) to actual tubes encounters some difficulties. As a first point in the derivation of (16), a plane cathode and grid are assumed and in the grid plane a homogeneous distribution of grid potential. Due to the grid-wire spacing and the action of positive-po-

[2] Refer to p. 683 of Bibliograghy reference.

tential electrodes the latter condition is by no means satisfied in tubes. Moreover, the distance between the grid surface and the cathode varies, as is seen from Fig. 4. Thus, practical conditions depart widely from those assumed in theory. It is difficult, if not impossible, to take even an approximative account of these practical conditions theoretically. As a second point, there is a potential minimum between cathode and grid because of the initial velocity (Maxwellian distribution) of the electrons drawn from the cathode. In theory, a zero initial velocity of electrons is assumed.

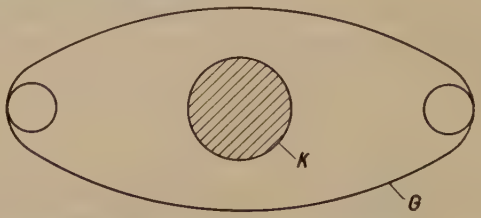

Fig. 4—Sketch of the cathode-grid arrangement of a commercial valve (K = cathode, G = grid).

The above points make a quantitative application of the electron-transit formula (16) to actual measurements impossible. A qualitative application will be tried here. Taking the second experiment of Section V, the cathode and the grid were both oval in shape, the cathode being 1.2×2.0 millimeters in cross section with an emitting layer of 75 microns. The grid cross section is 2.0×4.2 millimeters (outside dimensions) and the grid-wire diameter is 60 microns. The cathode-grid distance d is about 0.3 millimeter. The electron-transit time τ in seconds is $\tau = 0.51 \cdot 10^{-7}\ d\sqrt{V}$, where V is the grid-surface potential in volts. Calculating this value from $2/3\ SV = i_a$, where the conductance S is 6 milliamperes per volt and the anode current $i_a = 4$ milliamperes, we find $V = 1$ volt and hence $\tau = 1.5 \cdot 10^{-9}$. Thus, using (16), $R_{el} = 17{,}000$ ohms, whereas the measured value is about 7000 ohms. The large difference must be ascribed to the points considered above and, moreover, to the influence of the transit time between the input grid and the screen grid on input loss which was adequately dealt with by North.[20] Applying this correction yields a calculated value of $R_{el} = 9000$ ohms.

The influence of the input-grid—screen-grid space has been investigated in some measurements. Taking the valve AF7 the anode tension was 250 volts, the screen-grid tension was varied and the input-grid bias adjusted such that a constant cathode direct current i_k was obtained at varying screen-grid tensions. This results in an approximately constant grid-surface potential and hence constant transit time τ. We

[20] Refer to Bibliography.

measured input parallel resistance R_e at 16.3 meters wavelength and the difference ΔC_i between the cold-tube capacitance and the actual input capacitance. The suppressor was earthed.

Values of R_e were

V_{g2} (volts)	$i_k=1$ ma	$i_k=2$ ma	$i_k=3$ ma	$i_k=4$ ma
50	$43\cdot10^3$ ohms	$32.5\cdot10^3$ ohms	—	—
100	$62\cdot10^3$ ohms	$52\cdot10^3$ ohms	$48\cdot10^3$ ohms	$45\cdot10^3$ ohms
150	$80\cdot10^3$ ohms	$62\cdot10^3$ ohms	$58\cdot10^3$ ohms	$54\cdot10^3$ ohms
200	$98\cdot10^3$ ohms	$74\cdot10^3$ ohms	$65\cdot10^3$ ohms	$61\cdot10^3$ ohms
300	$138\cdot10^3$ ohms	$95\cdot10^3$ ohms	$78\cdot10^3$ ohms	$72\cdot10^3$ ohms

Values of ΔC_i were (micromicrofarads)

V_{g2} (volts)	$i_k=1$ ma	$i_k=2$ ma	$i_k=3$ ma	$i_k=4$ ma
50	1.52 $\mu\mu$f	1.68 $\mu\mu$f	—	—
100	1.13 $\mu\mu$f	1.29 $\mu\mu$f	1.36 $\mu\mu$f	1.44 $\mu\mu$f
150	0.90 $\mu\mu$f	1.15 $\mu\mu$f	1.21 $\mu\mu$f	1.27 $\mu\mu$f
200	0.82 $\mu\mu$f	1.02 $\mu\mu$f	1.12 $\mu\mu$f	1.15 $\mu\mu$f
300	0.61 $\mu\mu$f	0.81 $\mu\mu$f	0.93 $\mu\mu$f	0.98 $\mu\mu$f

The values of R_e increase somewhat faster than proportional to $(V_{g2})^{\frac{1}{2}}$. The input-grid-bias values were

i_k(ma)	$V_{g2}=100$ (volts)	$V_{g2}=200$	$V_{g2}=300$
1	$-V_i=3.28$ (volts)	7.10 volts	11.20 volts
2	$-V_i=2.76$ (volts)	6.42 volts	10.40 volts
3	$-V_i=2.30$ (volts)	5.96 volts	9.84 volts
4	$-V_i=1.92$ (volts)	5.52 volts	9.38 volts

In these measurements it was not possible to separate the influence of potential minimum, unhomogeneous potential on the grid surface, and transit time between the input grid and screen grid. It is, however, undoubtedly shown, that the latter cause has a considerable influence in valves of this type.

Mainly for applications to hexode-type high-frequency amplifier and mixer tubes, the influence of the suppressor-grid tension on input loss and input capacitance was investigated. A bias tension on the suppressor grid causes part of the electron current, flowing through the screen grid, to return. Some of the returning electrons arrive again in the vicinity of the input grid and add to the input losses. This state of things is to be considered as more or less normal in hexodes used as high-frequency amplifiers and mixers. With the AF3 valve the anode voltage was $V_a=250$ volts, $V_{g2}=100$ volts, and $V_{g1}=-3$ volts. We have measured input parallel resistance R_e and input capacitance C_e at 22.7 meters wavelength.

$-V_{g3}$ (volts)	1	5	10	15	20	25	30	35	40	47.5
R_e (k ohms)	111	108	105	100	92	79	63	47	34	29
C_e ($\mu\mu$f)	7.8	7.8	7.82	7.86	7.89	7.92	8.05	8.17	8.36	8.56
i_a (ma)	7.72	7.68	7.60	7.40	7.20	6.85	6.28	5.25	3.73	1.25

At 8.7 meters wavelength we found

$-V_{g3}$ (volts)	1	5	10	15	20	25	30	37.5
R_e (k ohms)	15.3	15.0	14.8	14.0	13.0	11.7	9.6	6.8

These measurements show that variations of $1/R_e$ and of C_e are approximately proportional to variations of i_a until the latter values exceed 4 milliamperes.

A simple calculation, which is omitted for the sake of briefness, shows,[2] that theory can only explain a small part of the additional loss due to returning electrons. Assuming, however, that a considerable number of electrons return more than once, this discrepancy is readily understood. This effect is dealt with in a forthcoming paper.

VII. Measurements on Anode Admittance and on Feed-Back Admittance

It follows from the formulas of Sections II and III that anode admittance and feed-back admittance are essentially caused by inductive effects in the short-wave region.

As to anode impedance, this is strongly influenced by small mutual inductances between the input-grid lead and the anode lead. In receivers as well as in anode-admittance measuring devices, a most careful screening is necessary. It is not to be expected, because of these inductive effects, that measuring devices of different construction yield identical values for output admittance and for feed-back admittance of the same valve. These effects are, of course, also present in measuring devices for the input admittance, but here the measurement is considerably simplified by the fact that active input parallel resistance is about ten times smaller than active output parallel resistance. Spurious effects of parts of the measuring-device chassis are relatively less important with input-admittance measurements.

Considering the measured values of active anode parallel resistance as a function of ω for any individual value, it appears that they are inversely proportional to ω^2, as is expected by theory. If these values of R_a (active) were principally due to electron-transit times, the same dependence on ω^2 would be expected. We have, however, a simple means for investigating the cause of R_a (active) by considering its dependence on the mutual conductance.

Measurements were carried out for several valves on 8.0 meters, including the types AF3 and AF7. The slope was varied in these experiments by applying suitable bias tensions to the input grid. The dimensions, capacitances, and lead lengths of the types AF3 and AF7 are equal, except for the first grid, which is of the variable-μ type

[2] Refer to Bibliography.

for the AF3 and of the constant-μ type for the AF7. The curve R_a (active) against grid bias is very different for these two valves. Fig. 5 shows $1/R_a$ (active) as a function of the mutual conductance for the valve AF3. It is a straight line, as it should be by the theory (see (8)). From this curve the magnitude of induction coefficients causing the active resistance may be calculated. We use the simplified formula $1/R_a$ (active) $= \omega^2 SCM$, where S is the mutual conductance, C the capacitance, and M a coefficient of mutual induction. Only one single capacitance and one single inductance coefficient is here assumed. Taking the values of Fig. 5 we find $CM = 8.5 \cdot 10^{-20}$ and with a capacitance C of $3.5 \cdot 10^{-12}$ farad this yields $M = 24 \cdot 10^{-9}$ henry. This is a value, fitting rather well in the order of magnitude, following from the equations of Section IV.

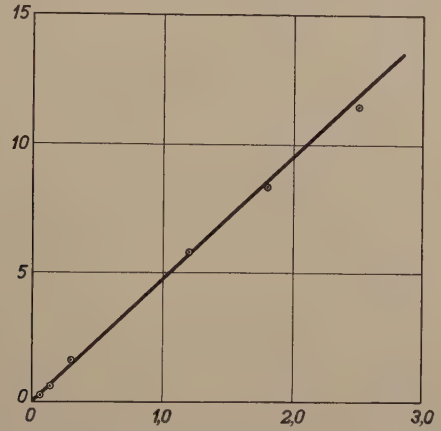

Fig. 5—Vertical axis: Values of $1/R_a$ (active) in mhos multiplied by 10^6 for the valve AF3. Horizontal axis: mutual conductance of this valve in milliamperes per volt, varied by putting bias tension on the input grid. The linear relation, demanded by the present theory, is satisfied by the measured values. The wavelength is 8.0 meters.

The influence of the tube dimensions (leads and capacitances) is shown by measurements on the types AF3 and AF7, compared with the types EF5 and EF6, the latter two having identical mutual conductances with the former two, but much smaller dimensions. Active anode parallel resistance is considerably greater for the latter two types in the short-wave region.

A valve type as measured, deviating in several respects from those mentioned above, is shown below.

Valve	Wavelength meters	R_a (biased) megohms	R_a (normal) megohms	R_a (active) megohms
4673	21.0	0.19	0.14	0.53
4673	10.9	0.097	0.065	0.20
4673	6.3	0.047	0.030	0.08

The valve 4673 (Philips) is a high-mutual-conductance pentode, designed as a high-frequency amplifier for television purposes (mutual conductance about 5 milliamperes per volt at 8 milliamperes anode direct current). R_a (active) is about the same for this valve as for the valves AF3 and AF7, having less than half its mutual conductance. We have obtained this result by applying a favorable sequence of the leads in the valve pinch. Ordinarily, the cathode lead is adjacent to the suppressor-grid lead. With the 4673, however, the cathode lead was placed between the heater leads. The capacitance C_{ak}, together with the coefficients M_{ak}, M_{mk}, M_{g3k}, and M_{g2k} are small for the 4673.

Finally, an anode-resistance measurement is given, wherein electron-transit times are manifest. Some valves of the type EF5 (variable-μ high-frequency pentode) were fitted with an anode of gauze material instead of massive plate. On 6.2 meters the results were

Valve	R_a (biased) megohms	R_a (normal) megohms	R_a (active) megohms
EF5 (normal)	0.155	0.064	0.109
EF5 (gauze anode)	0.142	0.038	0.052

As no other differences in the construction of these valves are known, the decrease of R_a (active) must be due to electrons crossing the anode and returning to it after a long way.

As to feed-back admittance, the frequency dependence and the independence of mutual conductance show that measured values are in accord with theory.[26] The influence of self- and mutual inductances of receiver chassis parts on the feed-back admittance may be illustrated by considering a self-inductance L, common to the input grid and the anode side. The quantity A of the theoretical (and experimental) formula

$$Y_{ag}' = j\omega(C_{ag} - A\omega^2)$$

is in this case altered into

$$A' = A + (C_0 L C_e) \cdot 10^{12}.$$

Here C_0 is the output and C_e the input capacitance of the valve. Taking L only $5 \cdot 10^{-9}$ henry and C_0 and C_e both 10 micromicrofarad, we get $A' = A + 50 \cdot 10^{-20}$, whereas A is about $70 \cdot 10^{-20}$. Hence an inductance of $5 \cdot 10^{-9}$ henry, which, according to Section IV must be considered as extremely small, may already cause a very considerable variation of A.

From the equations of Section III it may be deduced that A may be decreased considerably by a suitable mutual inductance between

[26] Refer to Bibliography.

the anode lead and the screen-grid lead of a high-frequency pentode. A valve of type EF5 was fitted with 7/4 turns in the anode lead and 1 turn in the screen-grid lead (diameter of turns about 13 millimeters) close together. Feed-back impedance on 5.1 meters was increased from 0.058 (normal value) to 0.5 megohm. This improvement of feed-back impedance, as it affects A, holds good for the whole short-wave range.

As to mutual conductance at short waves, an extended series of measurements has been carried out, but results need more discussion than would fit into the scope of this article.[29] It may be observed, however, considering (5a), (7), and (12), that the phase angle of mutual conductance due to inductive effects is very small, for instance some degrees at 5 meters with normal high-frequency valves.

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CHARACTERISTICS OF THE IONOSPHERE AT WASHINGTON, D. C., JUNE, 1938*

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DATA on the critical frequencies and virtual heights of the ionosphere layers for June, 1938, are given in Fig. 1. Fig. 2 gives the maximum frequencies which could be used for radio sky-wave communication by way of the normal E, F, F_1, and F_2 layers. The flatness of the graphs indicates very little percentage change in the maximum usable frequencies from day to night. This is characteristic of the months around the summer solstice.

Because of reflections from clouds of sporadic E layer during frequent irregular periods, the maximum usable frequencies often exceeded the regular, dependable values shown in the graphs. Occasionally they even exceeded 60 megacycles. Because of the erratic occurrence of these reflections, they could not be included in the average graphs.

The ionosphere storms which occurred during June are listed in Table I approximately in the order of their severity.

TABLE I
Ionosphere Storms (approximately in order of severity)

Date and hour E.S.T.	h_F before sunrise, km	Minimum $f_F{}^x$ during day (before sunrise), kc	Noon $f_{F_2}^x$ kc	Magnetic character[1]		Ionosphere character[2]
				00–12 G.M.T.	12–24 G.M.T.	
June 8	306	5100	6200	1.2	1.1	1
June 12	304	5200	6300	0.8	1.1	1
June 13 until 0900	338	4800	—	1.3	0.8	½
June 10 after 2100	—	—	—	0.5	0.6	½
June 11 until 0600	312	4500	—	0.6	0.6	½
June 22 until 1700	302	5000	6900	0.5	0.0	½
Average for undisturbed days	291	5800	8100	0.2	0.2	0

[1] American magnetic-character figure, based on observations of seven observatories.
[2] An estimate of the severity of the ionosphere storm at Washington on an arbitrary scale 0, ½, 1½, and 2, the character 2 representing the most severe disturbance.

Recent observations given in this series of reports have led to the following tentative picture of an ionosphere storm. The ionosphere storm, like the magnetic storm, has two phases. The first, or violent

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phase, occurs principally in the auroral zone. The second, or moderate phase, is much longer in duration and spreads out from the auroral zone toward the equator.

The violent phase consists of a boiling or turbulence of the entire ionosphere in the auroral zone, resulting in irregularly moving small clouds of ionization. The stratification of the ionosphere from the E

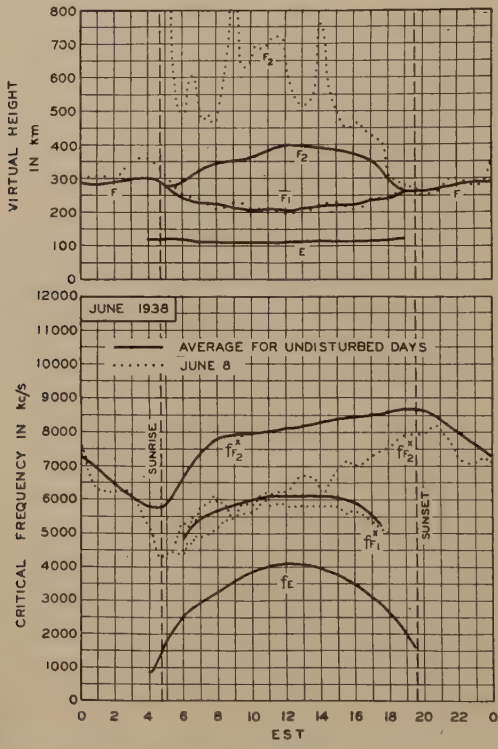

Fig. 1—Virtual heights and critical frequencies of the E, F, F_1, and F_2 layers of the ionosphere for June, 1938. The solid-line graphs represent the average for undisturbed days. The dotted graphs represent values for the ionosphere storm day of June 8.

layer up is literally torn to pieces. Either no reflections at all or very erratic reflections are observed during this period. This phase only occasionally extends as far south as Washington. On several occasions when the auroral zone did extend as far south as Washington, an increase of F-layer ionization has been observed to precede the violent phase.

The second and more moderate phase extends to the latitude of Washington much more frequently than the violent phase. This phase

is characterized by an expansion and diffusion of the higher F region. This expansion and diffusion result in abnormally low critical frequencies and large virtual heights of the night F and daytime F_2 layers and to a lesser extent of the daytime F_1 layer. The daytime E layer is not appreciably affected during this phase of the storm. Increased absorption is especially noticeable on the x components of the daytime F_1 and F_2 layers. The ionosphere storms reported for June were all of the moderate phase.

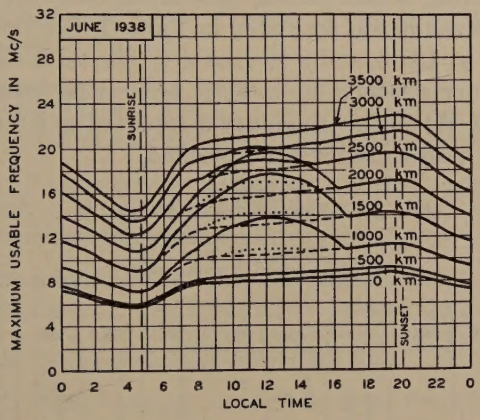

Fig. 2—Maximum usable frequencies for radio sky-wave transmission, June, 1938. Solid-line graphs represent average values for undisturbed days, for dependable transmission by the regular ionosphere layers. The values shown were considerably exceeded during frequent irregular periods by reflections from clouds of sporadic E layer. For distances of 1000, 1500, and 2000 kilometers, the dotted and dashed portions of the graphs represent maximum usable frequencies for F_1- and F_2-layer transmission, respectively, when these were less than those determined by the E layer. For distances of 2500 and 3000 kilometers the dashed line represents maximum usable frequencies for F_2-layer transmission when these were less than those determined by the F_1 layer.

Table II shows the number of hours the night f_F^z and daytime $f_{F_2}^z$ differed from the average for the undisturbed days of June by more than the given percentage. The disturbed hours are those listed in Table I.

TABLE II

CRITICAL FREQUENCY VARIATION FOR 720 HOURS OF OBSERVATIONS

Per cent	−30	−20	−10	−0	+0	+10	+20
Number of hours	1	30	115	391	329	48	3
Disturbed hours	1	26	65	83	3	0	0
Undisturbed hours	0	4	50	308	326	48	3

Sudden disturbances of the ionosphere as evidenced by radio fade-outs observed at Washington during June are listed in Table III.

TABLE III
SUDDEN IONOSPHERE DISTURBANCES

Date	G.M.T.			Location of transmitter	Minimum[1] relative intensity
	Beginning of fade-out	Beginning of recovery	Recovery complete		
June 29	1505	1510	1520	Ontario, Mass., D.C.	0.0
June 30	1504	1514	1535	Ontario, Mass., D.C.	0.0

[1] Minimum relative intensities are in terms of received intensity from CFRX, 6070 kilocycles, 600 kilometers.

The days during which strong sporadic-E reflections were most pronounced at Washington are listed in Table I. Continuous observations were made at 4.5, 6, and 8 megacycles. The table shows the hours of these days during which sporadic-E reflections were observed at these frequencies. When the frequency is reported as 8 megacycles, this value may have been considerably exceeded.

TABLE IV
SPORADIC-E FREQUENCIES IN MEGACYCLES

Midnight to noon
360 hours of observation

Date	Hour, E.S.T. 00	01	02	03	04	05	06	07	08	09	10	11
June												
5			8	8	8	8	8	8	4.5	8	8	8
6	6	4.5	4.5	4.5	4.5		8	4.5	4.5	4.5	4.5	
7				4.5			4.5	8	8	8	4.5	4.5
9				4.5	4.5	6	6	6	6	6	4.5	8
18	8	8	8	8	8	4.5		8	8	8	8	4.5
19	8	8	8	8								
20			8	8	8	8		4.5				
22	4.5	8	8	8	4.5							
28	8	4.5	8	4.5				4.5	8	4.5	4.5	
30	4.5	4.5	8	8	8	4.5	6	4.5	4.5	4.5	4.5	4.5

Noon to midnight
360 hours of observation

Date	Hour, E.S.T. 12	13	14	15	16	17	18	19	20	21	22	23
June												
3						8	8	8	8	4.5		
4								8	8	8	8	8
5						8	8	8	8	6	8	8
6					4.5	8	8	8	6	8		
10		4.5	8	8	8	4.5						
17							4.5		4.5	8	8	8
18	4.5	4.5	4.5			4.5	4.5	8	8	8	4.5	8
27	8	4.5	6			4.5	4.5	8	8	8	8	8
29								4.5	8	8	4.5	6

BOOKLETS, CATALOGS, AND PAMPHLETS RECEIVED

The following commercial publications of radio engineering interest have been received by the Institute. You can obtain a copy of any item without charge by addressing the issuing company and mentioning your affiliation with the Institute of Radio Engineers.

AIR-COOLED TRANSMITTING TUBES • • • *RCA Manufacturing Company, Inc., Harrison, N. J. Bulletin TT-100, 16 pages, 5½×8½ inches, lithographed.* Gives tabular operating data.

ASSEMBLY EQUIPMENT • • • *The George L. Hall Company, Old Lyme, Conn. Folder, 4 pages, 8½×11 inches, printed.* Description of pan-type and hopper-type bins designed to speed-up assembly operations involving small parts.

5-KW BROADCAST TRANSMITTER • • • *Graybar Electric Company, 420 Lexington Avenue, New York, N. Y. Bulletin T1477, 16 pages, 8×11 inches, printed.* Description of Western Electric 405A type.

CAPACITANCE BRIDGES • • • *General Radio Company, Cambridge A, Mass.* Experimenter, *for June, 8 pages, 6×9⅛ inches, printed.* Description and operating characteristics of 2 new 60-cycle bridges for capacitance and power-factor measurements on condensers and dielectric samples.

COILS • • • *J. W. Miller Company, 5917 South Main Street, Los Angeles, Calif. Catalog No. 39, 32 pages+cover, 8½×11 inches, printed.* The company's complete line of radio-frequency and intermediate-frequency coils, transformers, and chokes.

LACQUER • • • *Communication Products, Inc., 245 Custer Avenue, Jersey City, N. J. Bulletin 21, 4 pages, 3½×6¼ inches, printed.* Improvements in Q-Max lacquer, protective coating medium for radio-frequency products.

POWER RHEOSTAT • • • *International Resistance Company, 401 North Broad Street, Philadelphia, Pa. Bulletin VI, 4 pages, 8½×11 inches.* Description and operating data on a 25-watt rheostat of novel design. Included are performance curves showing temperature rise as function of load and the fraction of the winding in service.

VIBRATION PICKUPS • • • *Shure Brothers, 225 West Huron Street, Chicago, Ill. Data Sheet No. 163, 4 pages, 8½×11 inches, lithographed.* Covering description, characteristics and operating instructions on 2 piezo-electric vibration pickups. Output voltage is proportional to acceleration but modifying networks to produce voltages proportional to velocity or displacement are described.

VOLUME CONTROL • • • *International Resistance Company, 401 North Broad Street, Philadelphia, Pa. Bulletin V, 4 pages, 8½×11 inches.* The switch on this new attenuator is a molded motor commutator with a multi-finger brush.

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